METHODS OF PARTIAL DIFFERENTIAL EQUATIONS  
MATH-4500  
Assignment #4  
Due: March 27, 2001

1. (a) Find the temperature distribution along a thin rod, whose lateral surface is insulated over the finite interval $I = \{x|0 < x < 1\}$. Suppose the boundary $x = 0$ is held a fixed temperature 0, boundary $x = 1$ is up against a sinusoidal temperature reservoir, and there is no external heat source. That is, solve

$$ u_t = ku_{xx}, $$

with the initial condition

$$ u(x,0) = 0, $$

and the boundary conditions

$$ u(0,t) = 0 \text{ and } u(1,t) = \sin t. $$

(b) (MAPLE) When $k = 1/10$, generate an animated solution for $u(x,t)$, and plot the animated sequence for $0 < t < 5$. *Suggestion:* Make use of the example programs in section 8.4 of the reference book by Articolo.

(c) Give the form of the limiting behavior of the solution as $t \to \infty$.


3. (a) Text: p. 158, #3
(b) Text: p.164, #3

4. Text: p.167, #2. *Hint:* What happens if $h$ is a positive integer?

5. (a) Text: p. 158, #1. What is the solvability condition and its physical interpretation?
(b) Solve Laplace’s equation in the same rectangle as in part (a), now with the boundary conditions

$$ u_x = g(y) \text{ on } x = 0 \quad u_x = 0 \text{ on } x = a $$

$$ u_y = f(x) \text{ on } y = 0 \quad u_y = 0 \text{ on } y = b, $$

where the appropriate solvability condition is assumed to be satisfied. *Hint:* Use part (a) and the fact that $f(x) = f_{av} + [f(x) - f_{av}]$, where $f_{av} = \frac{1}{a} \int_0^a f(x)dx$. 