

Exam 3 B

$$1) c_1 u + c_2 v + c_3 w = 0$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & a & 1 & 0 \\ 2 & a & 3 & 1 & 0 \\ -1 & -2 & 2 & 1 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & a & 1 & 0 \\ 0 & a-6 & 5 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array} \right]$$

a can be anything.

2) a) Same as Exam 3 A question 2 a)

$$b) Ax = b \text{ with } b = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} -7 & 2 & 4 \\ 4 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Test 3 B

3) Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -4 & 3 \end{bmatrix}$

a) Find A^{-1} .

$$A^{-1} = \frac{1}{3(2) - (5)(1)} \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}.$$

b) Find a matrix C satisfying $A^{-1}CA^t = BA^t$

$$AA^{-1}CA^t = ABA^t$$

$$CA^t = AB \cdot A^t$$

$$CA^t A^{-t} = ABA^t A^{-t}$$

$$C = AB$$

$$C = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 6 \end{bmatrix}$$

4) Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 & -2 & 0 \\ -1 & -2 & -1 & -1 & 4 & 1 \end{bmatrix}$$

a) Find a basis for the null space of A .

$$\left[\begin{array}{cccccc|c} 1 & 2 & 1 & 3 & -2 & 0 & 0 \\ -1 & -2 & -1 & -1 & 4 & 1 & 0 \end{array} \right] \xrightarrow{u} \left[\begin{array}{cccccc|c} 1 & 2 & 1 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 1 & 0 \end{array} \right] \xrightarrow{u} \left[\begin{array}{cccccc|c} 1 & 2 & 1 & 0 & -5 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1/2 & 0 \end{array} \right]$$

$$x_1 = -2x_2 - x_3 + 5x_5 + \frac{3}{2}x_6$$

$$x_4 = -x_5 - \frac{1}{2}x_6$$

Let $x_2 = r$
 $x_3 = s$
 $x_5 = t$
 $x_6 = u$

General vector: $\begin{bmatrix} -2r - s + 5t + \frac{3}{2}u \\ r \\ s \\ -t - \frac{1}{2}u \\ t \\ u \end{bmatrix} = r \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 3/2 \\ 0 \\ 0 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$

basis = $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3/2 \\ 0 \\ 0 \\ -1/2 \\ 0 \\ 1 \end{bmatrix} \right\}$

b) What is the rank of A ?

$$\text{Dimension} = \text{rank} + \text{nullity}$$

$$6 = \text{rank} + 4$$

$$2 = \text{rank}$$