

Exam 3A

$$1) \quad u = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \quad v = \begin{bmatrix} 2 \\ 0 \\ a \\ -2 \end{bmatrix} \quad w = \begin{bmatrix} -1 \\ a \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & a & 1 & 0 \\ 2 & a & 3 & 0 & 0 \\ -1 & -2 & 2 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & a & 1 & 0 \\ 0 & a-4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

thus $a \neq 4$

$$2) a) \quad A \begin{bmatrix} -7 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad A \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad A \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A \begin{bmatrix} -7 & 2 & 4 \\ 4 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{thus } \begin{bmatrix} -7 & 2 & 4 \\ 4 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix} = A^{-1}$$

$$b) \quad Ax = b \quad \text{with } b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} -7 & 2 & 4 \\ 4 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Test 3A

3) Let $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ -4 & 3 \end{bmatrix}$

a) Find A^{-1} .

$$\left[\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array} \right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 5/3 & 1/3 & 0 \\ 1 & 2 & 0 & 1 \end{array} \right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 5/3 & 1/3 & 0 \\ 0 & 1/3 & -1/3 & 1 \end{array} \right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 5/3 & 1/3 & 0 \\ 0 & 1 & -1 & 3 \end{array} \right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

b) Find a matrix C satisfying $A^{-1}CA^t = BA^t$

$$AA^{-1}CA^t = ABA^t$$

$$CA^t = ABA^t$$

$$CA^t A^{-t} = ABA^t A^{-t}$$

$$C = AB$$

$$C = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -14 & 15 \\ -6 & 6 \end{bmatrix}$$

4) Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 & 1 \\ -1 & -2 & -1 & 4 & -1 \end{bmatrix}$$

a) Find a basis for the null space. Find all x such that $Ax = 0$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & -2 & 1 & 0 \\ -1 & -2 & -1 & 4 & -1 & 0 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 2 & 3 & -2 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 2 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 2 & 0 & -5 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array} \right]$$

Notice $x_1 = -2x_2 + 5x_4 - x_5$

$$x_3 = -x_4$$

Let $x_2 = r$

$$x_4 = s$$

$$x_5 = t$$

General vector:

$$\begin{bmatrix} -2r + 5s - t \\ r \\ -s \\ s \\ t \end{bmatrix} = r \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 5 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{basis} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

b) What is the rank of A ?

$$\text{Dimension} = \text{rank} + \text{nullity}$$

$$5 = \text{rank} + 3$$

$$2 = \text{rank}$$