EXAM 2 A

3) $\int_C e^{x+y}\, ds = \int_C f(x,y)\, ds$ from $(0,0) \to (2,1)$

parametrize by $\begin{cases} x(t) = 2t \\ y(t) = t \end{cases}$

thus $r(t) = 2t\hat{\imath} + t\hat{\jmath}$. and $r'(t) = 2\hat{\imath} + \hat{\jmath}$ with $\|r'(t)\| = \sqrt{5}$

$$\int_0^1 e^{3t}\sqrt{5}\, dt = \frac{e^{3t}}{3}\sqrt{5}\Big|_0^1 = \frac{\sqrt{5}}{3}(e^3 - 1).$$

4) $I = \oint_C 3\ln(y^2+x^2)\,dx.$

Using Green's theorem

$$I = -\iint_R \frac{\partial 3\ln(y^2+x^2)}{\partial y}\, dA = -\iint \frac{6y}{y^2+x^2}\, dA.$$

Using transformation to Polar Coordinates.

$$-\iint_R \frac{6\cancel{r}\sin\theta}{\cancel{r^2}}\, \cancel{r}\, dr\, d\theta = -\int_{\pi/6}^{\pi/3}\int_{\sqrt{3}}^{4\sin\theta\cos\theta} 6\sin\theta\, dr\, d\theta =$$

$$= \int_{\pi/6}^{\pi/3} (-24\sin^2\theta\cos^2\theta + \sqrt{3}\cdot 6\sin\theta)\, d\theta$$

$a = -24$

thus $b = 6\sqrt{3}$