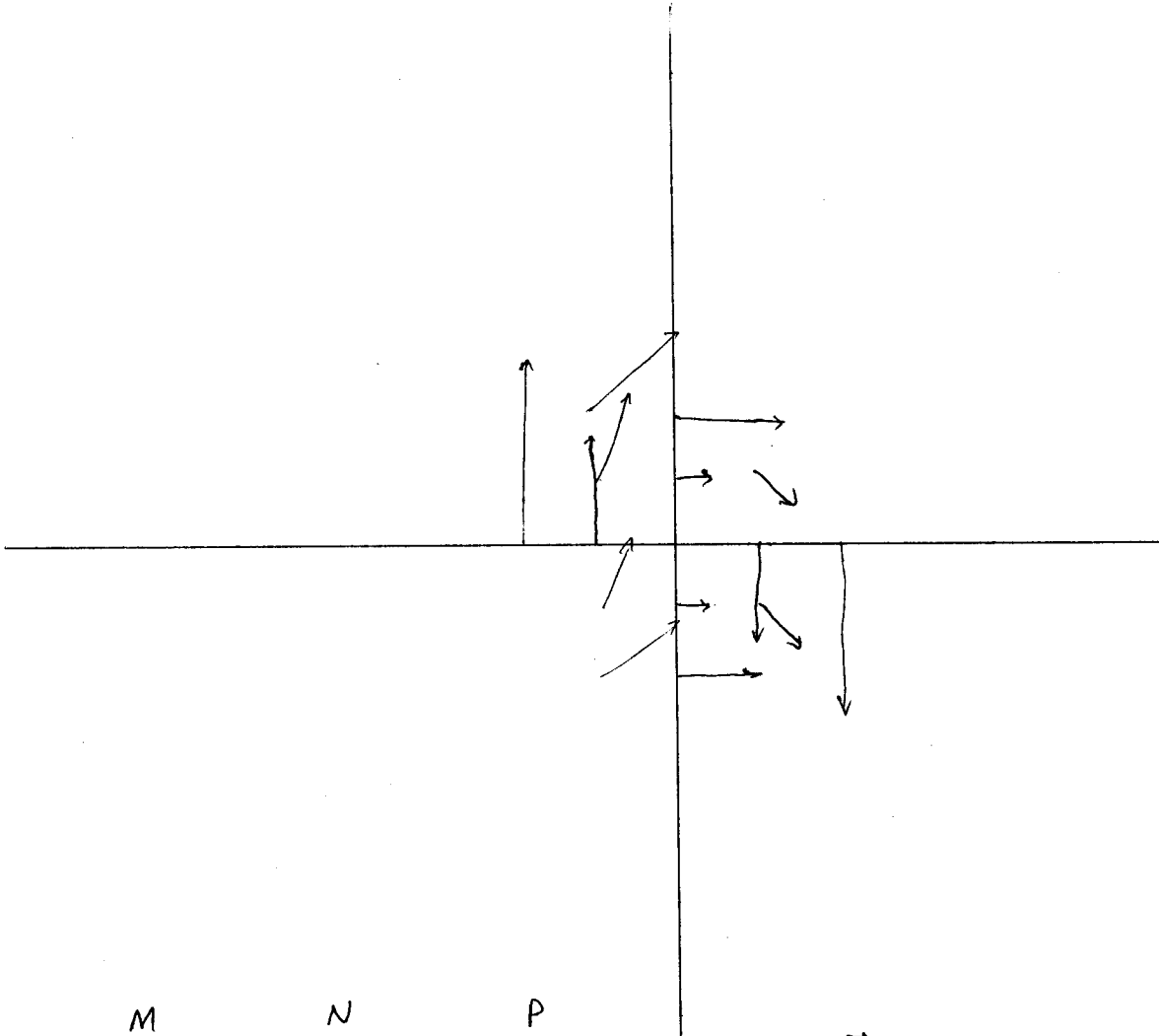


1) Test A

Sketch several representative vectors in the vector field

$$\vec{F} = y^2 \vec{i} - 3xz \vec{j}$$

You should draw at least two vectors in each quadrant, you should draw vectors at different distances from the origin, and you should draw some vectors that do not correspond to points on the axes.



2) (a) Let $\vec{F} = (4xy - z^2)\vec{i} + (2x^2 + 3z)\vec{j} + (3y - 2xz)\vec{k}$. Show that \vec{F} is conservative.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$$

$$\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$3 = 3 \checkmark$$

$$-2z = -2z \checkmark$$

$$4x = 4x \checkmark$$

$\therefore \vec{F}$ is conservative.

2) b) Test 4

Let C be the curve given by (t, t^2, t^4) for $0 \leq t \leq 1$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where \vec{F} was given in part (a)

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_0^1 \left[(4t^3 - (t^4)^2)\hat{i} + (2t^2 + 3t^4)\hat{j} + (3t^2 - 2t^4)\hat{k} \right] \cdot [1\hat{i} + 2t\hat{j} + 4t^3\hat{k}] dt \\ &= \int_0^1 (4t^3 - t^8 + 4t^3 + 6t^5 + 12t^5 - 8t^8) dt \\ &= \int_0^1 (8t^3 - 9t^8 + 18t^5) dt \\ &= \left[2t^4 - t^9 + 3t^6 \right]_0^1 \\ &= 2 - 1 + 3 = \underline{4}\end{aligned}$$

OR

Since it is conservative find f :

$$f = 2x^2y + 3yz - xz^2 + K$$

$$f(1, 1, 1) - f(0, 0, 0) = 2 + 3 - 1 = \underline{4}$$