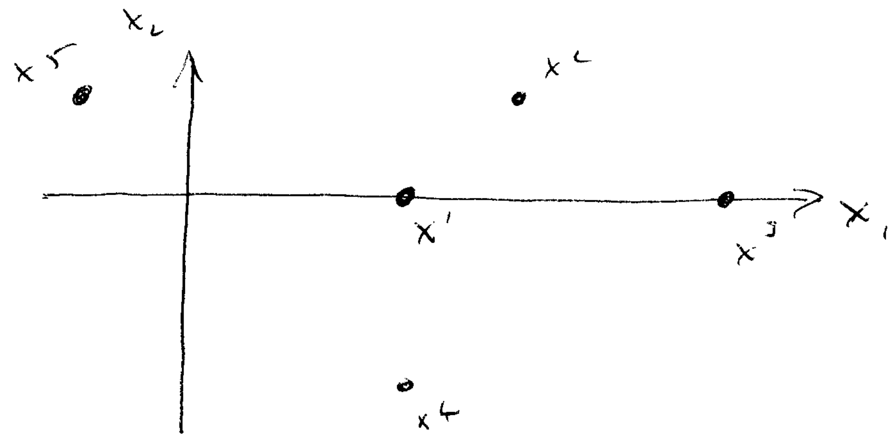
1. Let $y^1, y^2 \in AS$. Then $y^1 = Ax^1, y^2 = Ax^2,$
$x^1, x^2 \in S$

Let
$$z = \lambda y^1 + (1-\lambda) y^2, \quad 0 \le \lambda \le 1$$
$$= A(\lambda x^1 + (1-\lambda) x^2)$$
$$\in AS$$

2.



(a) $x^1 = \frac{1}{2}x^2 + \frac{1}{3}x^4 + \frac{1}{6}x^5 = \frac{2}{5}x^3 + \frac{1}{5}x^4 + \frac{2}{5}x^5$

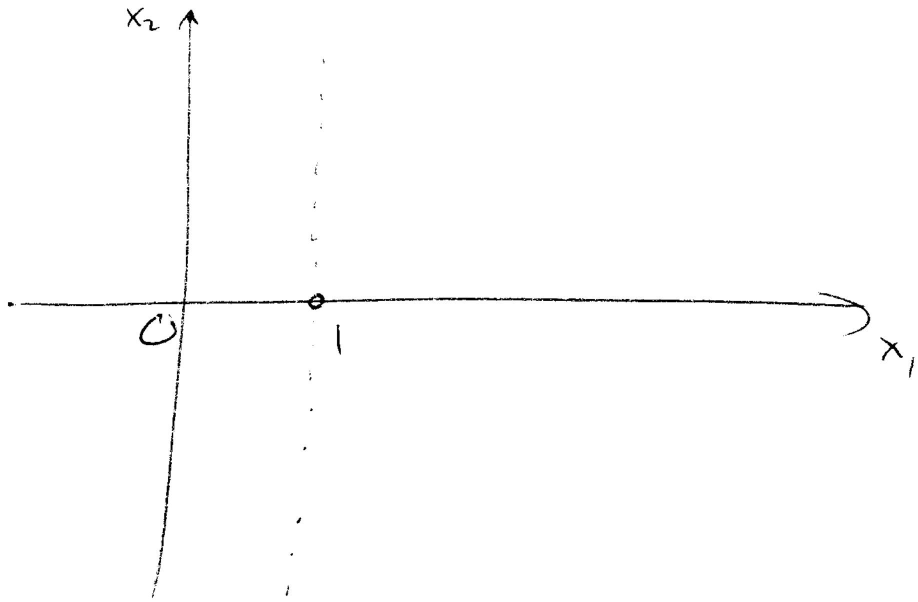
(b) $\operatorname{conv}(S) = \left\{ x \in \mathbb{R}^2 : \begin{bmatrix} 0 & 1 \\ -1 & -1 \\ 1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 0 \\ 5 \\ 10 \end{bmatrix} \right\}$

$$= \left\{ x \in \mathbb{R}^2 : \begin{array}{r} x_2 \leq 1 \\ x_1 + x_2 \geq 0 \\ x_1 + 2x_2 \leq 5 \\ 2x_1 - 3x_2 \leq 10 \end{array} \right\}$$

3.

$$S = \{x \in \mathbb{R}^2 : x_1 = 0 \text{ or } x = (1,0)\}$$

$$\operatorname{conv}(S) = \{x \in \mathbb{R}^2 : 0 \le x_1 < 1 \text{ or } x = (1,0)\}$$



4.

$x \in \text{conv}(S_1 \cap S_2)$

$\Rightarrow \quad x = \sum_{i=1}^{m} \lambda_i p^i \qquad \text{with} \quad p^i \in S_1 \cap S_2,$

$0 \leq \lambda_i \quad \forall i$

$\sum_{i=1}^{m} \lambda_i = 1$

$\Rightarrow \quad x = \sum_{i=1}^{m} \lambda_i p^i \qquad \text{with} \quad p^i \in S^j, \quad j = 1, 2$

$\Rightarrow \quad x \in \text{conv}(S^1) \text{ and } x \in \text{conv}(S^2)$

$\Rightarrow \quad x \in \text{conv}(S^1 \cap S^2).$

Eg. $S_1 = \{(1,0), (-1,0)\} \qquad S_2 = \{(0,1), (0,-1)\}$

$S_1 \cap S_2 = \phi, \qquad \text{conv}(S_1) \cap \text{conv}(S_2) = \{(0,0)\}.$

5.

$$\min_{x,t} \quad \sum_{i=1}^{m} t_i$$

$$\text{s.t.} \quad t_i \geq 0 \qquad \forall i$$

$$t_i \geq b_i - a^{i^T} x - a \qquad \forall i$$

$$t_i \geq -b_i + a^{i^T} x - a \qquad \forall i$$