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Name:

MATH2800 Introduction to Discrete Structures

First Exam, Tuesday, September 29, 2009.

You may use one sheet of handwritten notes, but no other sources. The exam consists of six questions, and lasts fifty minutes. Please work all six problems. Please show all work clearly and in reasonable detail. Answers without appropriate supporting work or requested explanations may not receive full credit. No calculators are allowed. Please ring your section below:

1: Monday 9am   2: Thursday 9am   3: Monday 2pm   4: Thursday 2pm

SOLUTIONS.

Q1	15	
Q2	10	
Q3	15	
Q4	15	
Q5	20	
Q6	25	
Total	100	

1. (15 points) In a certain town, 3/4 of the adult men are married to 4/7 of the adult women. Assume all marriages are monogamous (no one is married to more than one other person). Also assume there are at least 50 adult men in the town. What is the least possible number of adult men in the town? of adult women in the town? (Note: make sure that you show that your answer is the smallest consistent solution.)

Let $m = \#$ men, $w = \#$ women, $r = \#$ marriages

Then $r = \frac{3}{4} m$ and $r = \frac{4}{7} w$

$\Rightarrow m = \frac{4}{3} r = \frac{16}{21} w$

$\Rightarrow$ m is a multiple of 16

$\Rightarrow \boxed{m = 64} \Rightarrow r = 48 \Rightarrow \boxed{w = \left(\frac{7}{4}\right)(48) = 7 \times 12 = 84}$

2. (10 points) Find the greatest common divisor of 196 and 85.

$196 = 2 \times 98 = 2 \times 2 \times 49 = 2 \times 2 \times 7 \times 7$

$85 = 5 \times 17$

So $\gcd(85, 196) = 1$.

Alternatively, Euclidean Algorithm:

$196 = 2 \times 85 + 26$

$85 = 3 \times 26 + 7$

$26 = 3 \times 7 + 5$

$7 = 1 \times 5 + 2$

$5 = 2 \times 2 + 1$ So $\gcd(85, 196) = 1$.

3. (15 points) Find all integer solutions to the linear congruence

$$10x \equiv 5 \pmod{27}.$$

Find gcd $(27, 10)$:

$$\left.\begin{aligned} 27 &= 2\times 10 + 7 \\ 10 &= 1\times 7 + 3 \\ 7 &= 2\times 3 + 1 \end{aligned}\right\} \Rightarrow \begin{aligned} 1 &= 7 - 2\times 3 \\ &= 7 - 2\times(10 - 1\times 7) \\ &= 3\times 7 - 2\times 10 \\ &= 3\times(27 - 2\times 10) - 2\times 10 \\ &= \underbrace{3\times 27}_{=81} - \underbrace{8\times 10}_{=80} \checkmark \end{aligned}$$

So $-8\times 10 \equiv 1 \pmod{27}$.

So two ways:

(i) $-8\times 10\times 5 \equiv 5 \pmod{27}$,

or $10\times(-40) \equiv 5 \pmod{27}$

or $10\times(14) \equiv 5 \pmod{27}$

or $x = 14 + 27k$, k integer

(ii) $-8\times 10x \equiv -8(5) \pmod{27}$

or $+1x \equiv -40 \pmod{27}$

or $x = 14 + 27k$, k integer.

4. (15 points) Let $N(x)$ be the statement "x is from New York City", $R(x)$ be the statement "x has red hair", and $D(x)$ be the statement "x is taking Differential Equations this semester". The domain is the set of students in this class. Express the following statement in terms of $N(x)$, $R(x)$, and $D(x)$, quantifiers, and logical connectives:

   Every student in this class who is from New York City either has red hair or is taking Differential Equations this semester.

$$\forall x \left(N(x) \rightarrow \left(R(x) \vee D(x) \right) \right)$$

5. (20 points) Prove the following equality using induction:

$$\sum_{i=1}^{n+1} i2^i = n2^{n+2} + 2, \quad \text{for all integers } n \geq 0.$$

See Version A

6. (a) (10 points) Let A, B, and C be three sets. Is it true that $A \cap (B \cup C) = (A \cap B) \cup C$? Prove or give a counterexample.

   (b) (15 points) The symmetric difference of two sets A and B is defined as $A \oplus B := (A \cup B) - (A \cap B)$. Let A, B, and C be three sets. Assume $A \oplus B = A \oplus C$. Does it follow that $B = C$? Prove or give a counterexample.

See Version A.