TRUE/FALSE QUESTIONS

SETS & FUNCTIONS
Chapters #2 and #3

(1) A proper subset of a subset of a set, \( S \), is a proper subset of \( S \).

(2) The union of two non-empty sets is non-empty.

(3) The intersection of two non-empty sets is non-empty.

(4) If the set, \( A \), is a proper subset of the set, \( B \), the complement of \( A \) in \( B \) is non-empty.

(5) The empty set is a proper subset of the empty set.

(6) Every set is a subset of itself.

(7) If two sets are disjoint their union is the empty set.

(8) Let \( A \) denote a subset of the set \( U \). Then
\[ x \in U \setminus A \iff (x \in U) \land (x \notin A). \]

(9) Let \( A \) and \( B \) denote subsets of a set \( U \). Then
\[ (A \cup B)^c = A^c \cap B^c. \]

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(11) Let \( S \) and \( T \) denote non-empty sets. Let \( f : S \onto T \) and \( g : T \onto S \). Then \( f \) and \( g \) are both 1-1 functions.
(12) Let $S$ and $T$ denote non-empty sets. Let $f : S \rightarrow T$. Then the inverse function, $f^{-1}$, is defined on $T$.

(13) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. Then the domain of $f$ is $S$.

(14) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. Then the range of $f$ is $T$.

(15) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. Then the image of $S$ under $f$ is $T$.

(16) Let $S$ and $T$ denote non-empty sets. Let $f : S \rightarrow T$. Then $f^{-1}$ is an onto function.

(17) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. Let $B \subseteq T$. Then $f^{-1}(B) \neq \emptyset$.

(18) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. If $A$ is a subset of $S$ it follows that $A = f^{-1}(f(A))$.

(19) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3$, $\forall x \in \mathbb{R}$. Then $f^{-1}(x) = x^{\frac{1}{3}}$, $\forall x \in \mathbb{R}$.

(20) An onto function is necessarily one-to-one.

(21) Let $S$ and $T$ denote non-empty sets. Let $f : S \rightarrow T$. Then the two functions $f^{-1} \circ f$ and $f \circ f^{-1}$ are equal.

(22) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. If the inverse function, $f^{-1}$, exists, then necessarily $f$ is 1-1.

(23) Let $S$ and $T$ denote non-empty sets and let $f : S \rightarrow T$. If $x$ and $y$ are elements of $S$ such that $f(x) = f(y)$, then $x = y$.

(24) Let $S$ and $T$ denote non-empty sets. A function from $S$ into $T$ is a subset of the product space $S \times T$.

(25) Let $S$ and $T$ denote non-empty sets. Every subset of the product
space, $S \times T$, is a function from $S$ into $T$.

____(26) Let $A$ be a subset of the set $S$. For each such $A$ denote its characteristic function by $f_A$. Then if $A$ and $B$ are subsets of the set $S$ it follows that $f_{A \cup B} = f_A f_B$. 

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