

### Klee-Minty cube for $n=3$

$$0. \text{ min } -100x_1 - 10x_2 - x_3$$

$$\begin{array}{rcll} \text{s.t.} & \textcircled{x_1} & & \\ & 20x_1 + x_2 & + x_4 & = 1 \\ & 200x_1 + 20x_2 + x_3 & + x_5 & = 100 \\ & & & + x_6 = 10000 \\ & x_i \geq 0. & & \end{array}$$

1.  $x_1$  enters,  $x_4$  leaves:

$$\text{min } -10x_2 - x_3 + 100x_4 \quad -100$$

$$\begin{array}{rcll} \text{s.t.} & x_1 & & \\ & \textcircled{x_2} & & \\ & 20x_2 + x_3 & + x_4 & = 1 \\ & & -20x_4 + x_5 & = 80 \\ & & -200x_4 & + x_6 = 9800 \\ & x_i \geq 0 & & \end{array}$$

2.  $x_2$  enters,  $x_5$  leaves:

$$\text{min } -x_3 - 100x_4 + 10x_5 \quad -900$$

$$\begin{array}{rcll} \text{s.t.} & x_1 & & \\ & & \textcircled{x_4} & \\ & & -20x_4 + x_5 & = 80 \\ & & +200x_4 - 20x_5 + x_6 & = 8200 \\ & & & + x_3 = 1 \\ & x_i \geq 0 & & \end{array}$$

3.  $x_4$  enters,  $x_1$  leaves:

$$\text{min } 100x_1 - x_3 + 10x_5 \quad -1000$$

$$\begin{array}{rcll} \text{s.t.} & x_1 & & \\ & 20x_1 + x_2 & + x_4 & = 1 \\ & -200x_1 & & + x_5 = 100 \\ & & & + x_6 = 10000 \\ & x_i \geq 0 & \textcircled{x_3} & \end{array}$$

4.  $x_3$  enters,  $x_6$  leaves:

$$\begin{array}{rcll}
 \min & -100x_1 & & -10x_5 + x_6 & -9000 \\
 \text{s.t.} & \textcircled{x_1} & & & \\
 & 20x_1 + x_2 & +x_4 & +x_5 & = 1 \\
 & -200x_1 & +x_3 & -20x_5 + x_6 & = 100 \\
 & & & & = 8000 \\
 & & & & x_i \geq 0
 \end{array}$$

5.  $x_1$  enters,  $x_4$  leaves:

$$\begin{array}{rcll}
 \min & & & 100x_4 - 10x_5 + x_6 & -9100 \\
 \text{s.t.} & x_1 & & +x_4 & = 1 \\
 & x_2 & & -20x_4 \textcircled{+x_5} & = 80 \\
 & x_3 & & +200x_4 - 20x_5 + x_6 & = 8200 \\
 & & & & x_i \geq 0
 \end{array}$$

6.  $x_5$  enters,  $x_2$  leaves:

$$\begin{array}{rcll}
 \min & & 10x_2 & -100x_4 & +x_6 & -9900 \\
 \text{s.t.} & x_1 & & +x_4 & & = 1 \\
 & x_2 & & -20x_4 + x_5 & & = 80 \\
 & 20x_2 + x_3 & & -200x_4 & +x_6 & = 9800 \\
 & & & & & x_i \geq 0
 \end{array}$$

7.  $x_4$  enters,  $x_1$  leaves:

$$\begin{array}{rcll}
 \min & 100x_1 + 10x_2 & & & +x_6 & -10000 \\
 \text{s.t.} & x_1 & & & +x_4 & = 1 \\
 & 20x_1 + x_2 & & & +x_5 & = 100 \\
 & 200x_1 + 20x_2 + x_3 & & & +x_6 & = 10000 \\
 & & & & & x_i \geq 0
 \end{array}$$

Optimal.

Notice: if  $x_3$  had entered at Step 1 instead of  $x_1$ , then we would have solved the problem in one step.

The constraints are approximately the cube

$$\begin{aligned} 0 &\leq x_1 \leq 1 \\ 0 &\leq x_2 \leq 100 \\ 0 &\leq x_3 \leq 10000. \end{aligned}$$

We start at the origin, then (approximately) visit the vertices

- ①  $(1, 0, 0)$
- ②  $(1, 100, 0)$
- ③  $(0, 100, 0)$
- ④  $(0, 100, 10000)$
- ⑤  $(1, 100, 10000)$
- ⑥  $(1, 0, 10000)$
- ⑦  $(0, 0, 10000)$  - optimal.

