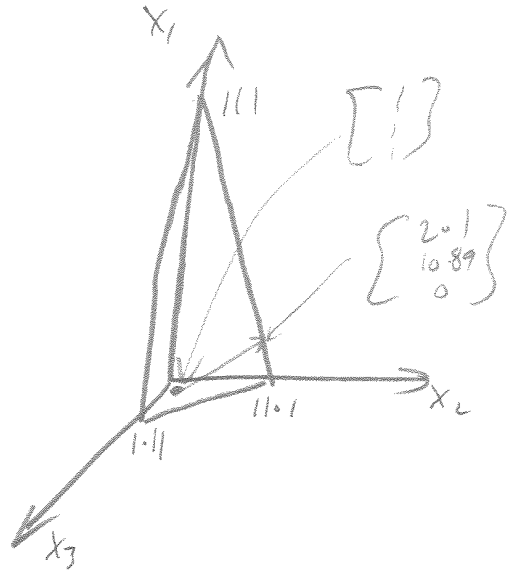
Projective Methods

The Contoring Direction



Eg: $\min \quad -x_1 - 9x_2$
$\text{s.t.} \quad x_1 + 10x_2 + 100x_3 = 111$
$x_i \geq 0.$

Optimal point: $x = [111 \;\; 0 \;\; 0]^T.$

Current feasible point: $x = \begin{bmatrix}1\\1\\1\end{bmatrix}$, so don't need to rescale.

Affine direction:

$A = [1 \;\; 10 \;\; 100) \qquad AA^T = 10101, \; (AA^T)^{-1} = \frac{1}{10101} \qquad Ac = 91$

$$-P_A c = -c + A^T(AA^T)^{-1}Ac = \begin{bmatrix}1\\9\\0\end{bmatrix} - \frac{91}{10101}\begin{bmatrix}1\\10\\100\end{bmatrix} \approx \begin{bmatrix}.99\\-8.9\\-.9\end{bmatrix}$$

$$\approx .9\begin{bmatrix}1.1\\-9.9\\-1\end{bmatrix}.$$

Moving in this direction, we get points of the form

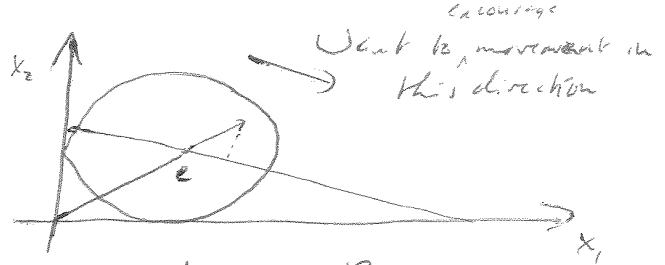
$x \approx \begin{bmatrix}1\\1\\1\end{bmatrix} + \alpha\begin{bmatrix}1.1\\-9.89\\-1\end{bmatrix}$, so taking $\alpha = 1$ gives the point $x \approx [2.1 \;\; 10.89 \;\; 0]^T$ on the bounding

So the direction does not move us ~~close~~ to appreciably closer to the optimal point.

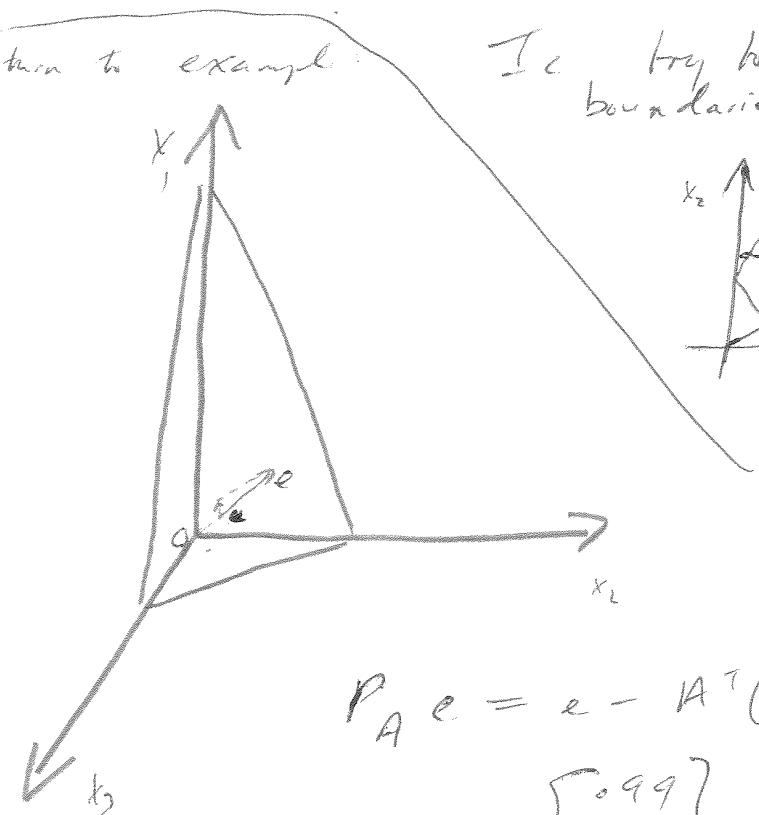
Direction: Draw a sphere around current point, find best point in sphere in feasible region.

Problems arise when the algorithm starts from near a vertex. So it is useful to try to "center" the iterates.

Return to example. Let try to move towards the "far-away" boundaries:



Look at $P_A e$.



$$P_A e = e - A^T(AA^T)^{-1}Ae = \begin{bmatrix}1\\1\\1\end{bmatrix} - \frac{111}{10101}\begin{bmatrix}1\\10\\100\end{bmatrix}$$

$$\approx \begin{bmatrix}.99\\.9\\-.1\end{bmatrix}$$

Does a better job of increasing x_1 than $-P_A c \approx \begin{bmatrix}1.1\\1.0\\-1\end{bmatrix}$.

In general:

<u>Defn</u> Given the LP $\min c^T x$, $Ax = b$, $x \geq 0$, and the feasible point $\bar{x} > 0$,

the <u>affine scaling direction</u> is $-\bar{X} P_{\bar{A}} \bar{c}$, and

the <u>centering direction</u> is $\bar{X} P_{\bar{A}} e$.