Why do we use probability to characterize anything about the real world?

Roughly to describe four kinds of uncertainty:

1) Aleatory uncertainty: describing outcomes of a repeated set of experiments
2) Epistemic uncertainty: describing outcomes of a deterministic process for which we have incomplete information.
3) Fundamental uncertainty: randomness is intrinsic to the system (quantum mechanics, Copenhagen interpretation)
4) Subjective uncertainty: actions of conscious beings

What does probability mean quantitatively? What does it mean to say that some event has a probability of 1/3 to happen?

1) Classical probability: give equal probability to all possible outcomes. Basic but limited in applicability.
2) Frequency-based interpretation of probability: The probability associated to an event is the fraction of times in which that event would be seen if the observed experiment were repeated many times under identical conditions. This might be OK even for experiments that are not repeated many times, as long as in principle a repeated experiment is possible. But really problematic is for fundamentally nonrepeatable situations: innovative industrial plan, historical analysis
3) Bayesian interpretation of probability: Probabilities are fundamentally subjective measures. But probability theory is a framework for rationally manipulating subjective models for uncertainty. Rough idea: declare all subjective assumptions about uncertainty (define probabilistic model), usually involving prior knowledge and initial conditions (prior distribution for the random variables). Then one can accumulate data and use known information about equations of motion, dynamics and there is a rational framework for updating the prior probability distribution to a posterior probability distribution which is influenced by the new information and dynamics of the system.

   We will not be too concerned with philosophical interpretations because we can basically sidestep philosophical interpretation by using Kolmogorov's axiomatic formulation of probability theory (in terms of measure theory).