DEFINITIONS

• function
• group
• field
• ordered field
• real numbers
• domain of a function
• codomain of a function
• onto function
• 1-1 function
• inverse function
• range of a function
• image of a pt. (set)
• preimage of a set
• composition of functions
• equal functions
• set union
• set intersection
• set complement
• countable set
• finite set
• uncountable set
• rational number
• irrational number
• integers
• upper bound of a set in $\mathbb{R}$ (and lower bound)
• least upper bound of a set in $\mathbb{R}$ (and glb)
• Archimedean property of $\mathbb{R}$
• subset
• proper subset
• graph of a function
• sequence
• finite sequence
• infinite sequence
• convergent sequence
• sequence converges to a real number
• sequence converges to infinity
• Cauchy sequence
• geometric series
• real-valued function continuous at a point
• absolute value function
• an infinite set
• the cardinality of a finite set
• two sets have the same cardinality
• the power set of a set
• the continuum hypothesis
• the Cantor set
• the ternary representation and decimal representation of an element in $[0,1]$. 
THEOREMS, WORKSHEETS and HOMEWORKS

- Two deMorgan’s Laws (proof, see Course Notes)
- Induction Theorem (statement and proof, see Course Notes)
- A non-empty set of integers, that is bounded below, contains a least element. (proof, see Course Notes)
- $|2^S| > |S|$ (cocktail party proof, see Course Notes)
- Decimal Representation Theorem (statement, see Course Notes)
- \[ f(A \cup B) = f(A) \cup f(B) \text{ and like statements} \] (proof, WS 3.1.2 & WS 3.3.1)
- In a group the zero element is unique and the inverse of an element is unique. (proof, WS 4.2.1)
- If \( x \in \mathbb{R} \) there exists \( n \in \mathbb{Z} \) such that \( n \leq x < n+1 \). (proof, WS 4.4.5)
- If \( f \) and \( g \) are real-valued continuous functions defined on some real domain, then so too is \( f + g \). (proof, WS 4.8.2)
- If \( f \) and \( g \) are real-valued continuous functions defined on some real domain, then so too is \( \lambda f, \forall \lambda \in \mathbb{R} \). (proof, WS 4.8.4)
- A finite set can have at most one cardinality. (proof, WS 5.1.1)
- Let \( \{a_k\}_{k=0}^{\infty} \) be a sequence in a field \( \mathcal{F} \). Prove that
  \[ \sum_{k=1}^{n} (a_k - a_{k-1}) = a_n - a_0, \forall n \in \mathbb{Z}^+. \]
  Such a sum is called a **telescoping sum**. (proof, WS 6.2.4)
- \( \mathbb{Z} \) is unbounded. (proof, HW 8)
- An infinite sequence in \( \mathbb{R} \) can have at most one limit in \( \mathbb{R} \). (proof, HW 10)
- Let \( \{a_k\}_{k=0}^{\infty} \) be a sequence in a field \( \mathcal{F} \) and let \( \lambda \in \mathcal{F} \). Prove that
  \[ \lambda (\sum_{k=0}^{n} a_k) = \sum_{k=0}^{n} \lambda a_k. \] (proof, HW 15)

TRUE/FALSE QUESTIONS

- all True/False questions from Chapters #2-7