Directions. Please submit your answer to the following problem in a \LaTeX-prepared document. Class participants are encouraged to prepare solutions in a collaborative mode but to prepare their to-be-submitted write-ups individually. The consequences of sharing files, electronic or otherwise, are discussed in the course syllabus.\footnote{If the wording of this problem was discussed in detail in the classroom, the course instructor expects to see similar phrases and sentences in reading the submissions.}

Please include the problem number along with a statement of the problem in your submission. Please also include your e-mail address.

**Problem.** Recall the following definition. Let \((M, d)\) denote a metric space and let \(\{x_n\}_{n=1}^{\infty}\) denote an infinite sequence in \(M\). A point \(x \in M\) is said to be an accumulation point of \(\{x_n\}_{n=1}^{\infty}\) if for every \(\epsilon > 0\) and for every \(N \in \mathbb{Z}^+\) there exists \(n > N\) such that \(d(x_n, x) < \epsilon\).

A. Provide a definition of the notion of a subsequence of \(\{x_n\}_{n=1}^{\infty}\). (Such a subsequence is frequently written \(\{x_{n_m}\}_{m=1}^{\infty}\).) (Your definition should permit the proof of part B, below.)

B. Let \(x\) denote an accumulation point of the sequence \(\{x_n\}_{n=1}^{\infty}\). Prove that there exists a subsequence of \(\{x_n\}_{n=1}^{\infty}\) that converges to \(x\).