Directions. Please submit your answer to the following problem in a LaTeX-prepared document. Class participants are encouraged to prepare solutions in a collaborative mode but to prepare their to-be-submitted write-ups individually. The consequences of sharing files, electronic or otherwise, are discussed in the course syllabus.\footnote{If the wording of this problem was discussed in detail in the classroom, the course instructor expects to see similar phrases and sentences in reading the submissions.}

Please include the problem number along with a statement of the problem in your submission. Please also include your e-mail address.

\textbf{Problem.} Let each rational number in \((0, 1)\) be expressed in the form \(\frac{p}{q}\) where \(p\) and \(q\) have no common integer divisors other than \(\pm 1\); that is \(\gcd (p, q) = 1\).

Define \(f : (0, 1) \to \mathbb{R}\) by

\[
f(x) = \begin{cases} 
\frac{1}{q} & \text{if } x = \frac{p}{q}, p, q \in \mathbb{Z}^+, \gcd (p, q) = 1 \\
0 & \text{if } x \notin \mathbb{Q}.
\end{cases}
\]

Prove that \(f\) is continuous at each irrational number in \((0, 1)\).