Directions. Please submit your answer to the following problem in a LaTeX-prepared document. Class participants are encouraged to prepare solutions in a collaborative mode but to prepare their to-be-submitted write-ups individually. The consequences of sharing files, electronic or otherwise, are discussed in the course syllabus.\footnote{If the wording of this problem was discussed in detail in the classroom, the course instructor expects to see similar phrases and sentences in reading the submissions.}

Please include the problem number along with a statement of the problem in your submission. Please also include your e-mail address.

**Problem.** Prove the following statements.

A. If $S$ is a non-empty subset of $\mathbb{R}$ and $S$ is bounded above, then there exists $x \in S$ such that $x + 1 \not\in S$.

B. Prove that the integers, $\mathbb{Z}$, as a subset of $\mathbb{R}$, is not bounded above.

C. Prove that for the positive numbers, $\epsilon$ and $M$, there exists $n \in \mathbb{Z}^+$, such that $n \epsilon > M$. (This is the Archimedean property of $\mathbb{R}$.)