Interior point methods for NLP:

LOQO, KNITRO, IPOPT

Can work with equality constraints.

To ease presentation, consider

$$\min f(x) \qquad f: \mathbb{R}^n \to \mathbb{R}$$
$$\text{s.t. } g(x) \le 0 \qquad g: \mathbb{R}^n \to \mathbb{R}^m \qquad f, g \text{ smooth.}$$

Barrier problem: $\min f(x) - \mu \sum \ln s_i$

$$\text{s.t. } g_i(x) + s_i = 0 \quad i = 1, \dots, m$$
$$s \ge 0$$

Optimality conditions:

$$\nabla f + A(x)^T y = 0 \quad (1)$$
$$-\mu S^{-1} e + Y e = 0 \quad (2)$$
$$g(x) + s = 0 \quad (3) \quad (\text{and } s \ge 0)$$

where $A(x)$ is Jacobian of $g(x)$,

and y are KKT multipliers

Replace (2) by $-\mu e + SYe = 0$

Applying Newton's Method to the system of equations:

Choose direction $(\Delta x, \Delta s, \Delta y)$ satisfying:

$$\begin{bmatrix} \nabla^2_{xx} L(x,s,y) & 0 & A(x)^T \\ 0 & Y & S \\ A(x) & I & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta s \\ \Delta y \end{bmatrix} = \begin{bmatrix} -\nabla f - A(x)^T y \\ \mu e - SYe \\ -g(x) - s \end{bmatrix}$$

Can eliminate Δs from this equation,

so ~~$\Delta s = -Y^{-1} S (\Delta y)$~~

$$\Delta s = -Y^{-1} S \Delta y + Y^{-1}(\mu e - SYe)$$

and then solve a system of the form:

$$\begin{bmatrix} \nabla^2_{xx} L(x,s,y) & A(x)^T \\ A(x) & -Y^{-1}S \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} \ell \\ D \end{bmatrix}$$

for appropriate vectors ℓ, D.

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Need a line search to pick step length.

Approach must be modified to cope with non-convexity

and to prevent convergence to non-stationary points.

Can be done either through a trust region approach,

or by modifying the primal-dual matrix.

For more details, see papers by Vanderbei et al,
and by Nocedal et al,
and by Wachter & Biegler

Wachter & Biegler (short) use filter linesearch,

and if converge to infeasible non-stationary point

then use a method to emphasize feasibility

(at expense of optimality).

Filter Methods (Fletcher & Leyffer)

Only accept a steplength if it improves either the objective or feasibility.