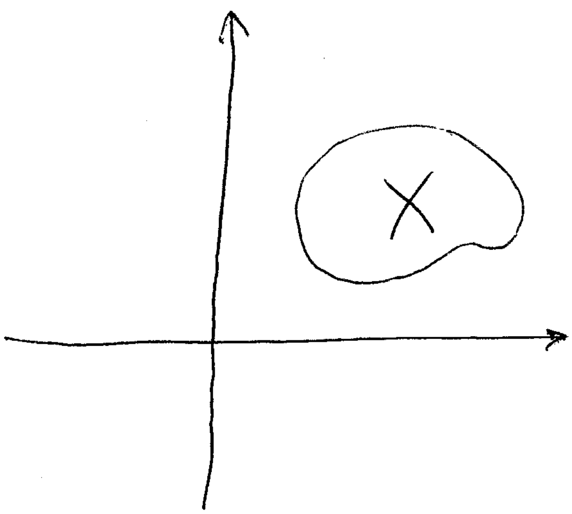
Geometric interpretation

(One inequality constraint)

$$\min f(x) \quad \text{s.t. } g(x) \le 0, \quad x \in X$$

$$f : \mathbb{R}^n \to \mathbb{R}$$
$$g : \mathbb{R}^n \to \mathbb{R}.$$

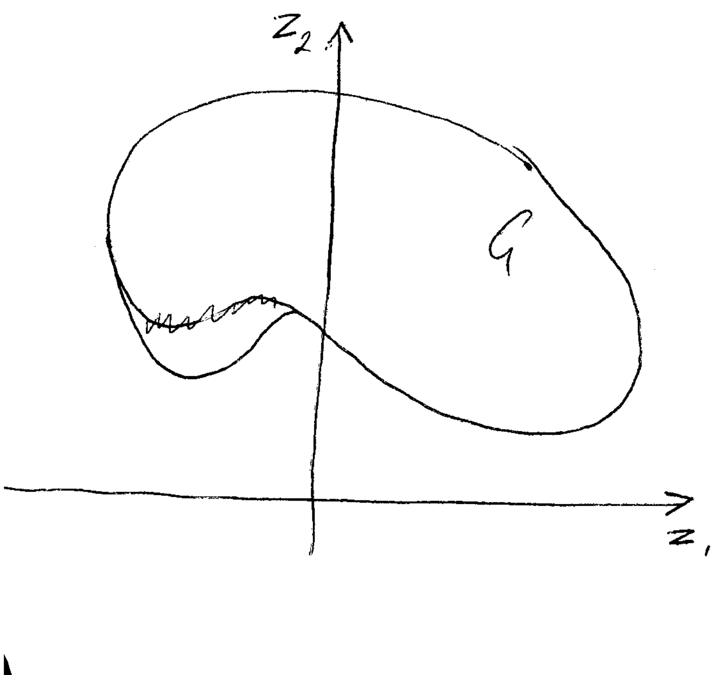
In x-space:



For each point $x \in X$, let

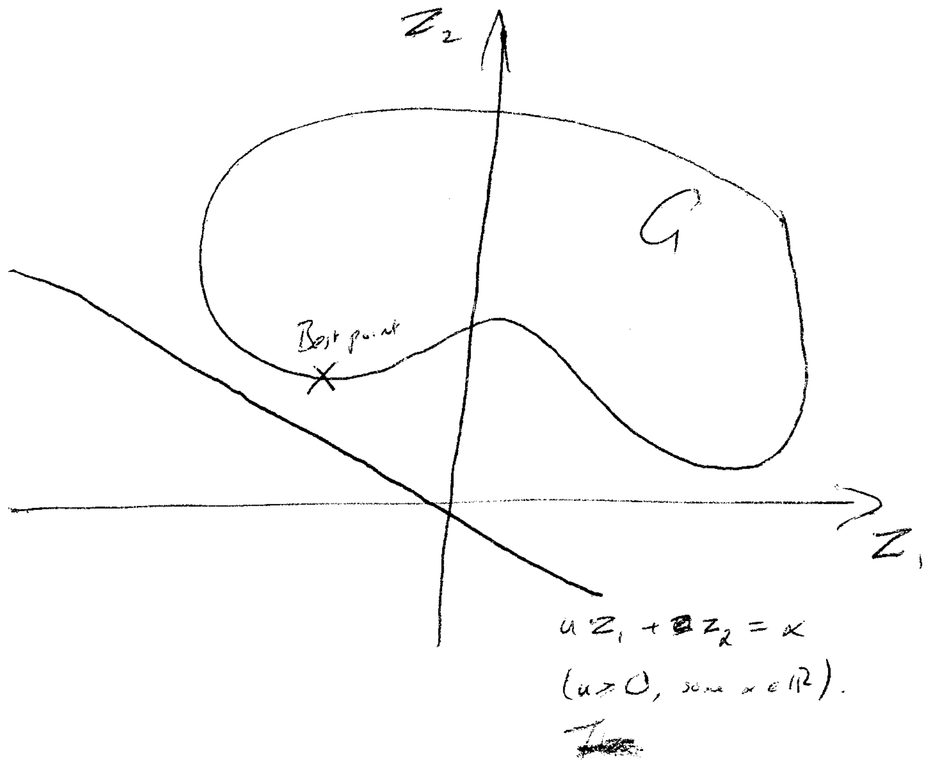
$$z_1 = g(x)$$
$$z_2 = f(x)$$

This defines a map from X to z-space:



Look at slices through G for fixed z_1.

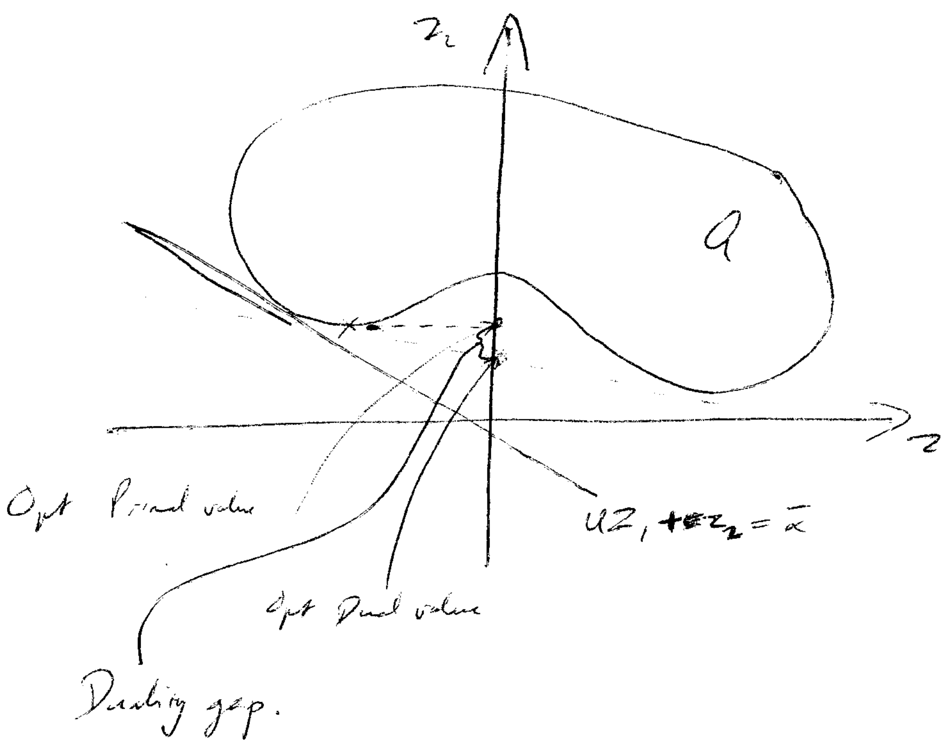
Each point in G corresponds to at least one point in X; every point in X corresponds to some point in G.



Need points to left of z_2-axis, since need $g(x) \leq 0$.

$$\min_{x \in X} \{ f(x) + u g(x) \}$$
$$= \min \{ z_2 + u z_1 : z \in G \}$$

Want to ~~make~~ find α so that $u z_1 + z_2$ is a supporting hyperplane for G

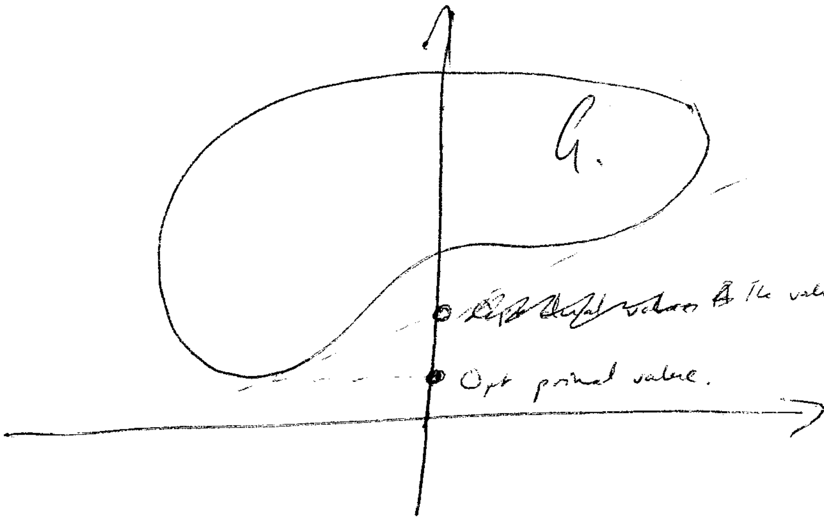


Notice that α is the value of the intercept on the z_2-axis

Dual problem is to maximize $\bar{\alpha}$ i.e. find u which gives highest intercept.
Then dual value is value of highest intercept.

NB: Why insist $u \geq 0$?

Note the following example:



Why not insist $v \geq 0$:

Consider $\min f(x)$

$h(x) = 0$

$x \in X$

$f: \mathbb{R}^n \to \mathbb{R}$, $h: \mathbb{R}^n \to \mathbb{R}$.

$H = \{(z_1, z_2):$
$z_1 = h(x), z_2 = f(x)$
for some $x \in X\}$.