Optimality Conditions for Nonsmooth Problems (§3.6 in Ruszczynski)

$$
\begin{aligned}
\min \quad & f(x) \\
\text{s.t.} \quad & g_i(x) \le 0 \quad i = 1, \dots, m \qquad \text{(NLP)} \\
& h_j(x) = 0 \quad j = 1, \dots, p
\end{aligned}
$$

Assume: $f(x)$, $g_i(x)$ $i = 1, \dots, m$ convex
$h_j(x)$ affine, $j = 1, \dots, p$.
f is smooth.
~~f and~~ g may be nonsmooth, continuous.

THM Let $\hat{x}$ be a local minimizer of (NLP). Assume Slater's constraint qualification holds. Assume also that a more restrictive constraint qualification (Robinson's) holds at $\hat{x}$.

Then there exist $\hat{u} \in \mathbb{R}^m_+$, $\hat{v} \in \mathbb{R}^p$, and subdifferentials $\xi_i(\hat{x})$ of g_i at $\hat{x}$ such that

$$\nabla f(\hat{x}) + \sum_{i=1}^{m} \hat{u}_i \, \xi_i(\hat{x}) + \sum_{j=1}^{p} \hat{v}_j \nabla h_j(\hat{x}) = 0$$

$$\hat{u}_i \, g_i(\hat{x}) = 0$$

The set of multipliers $\hat{u}(\hat{x})$, $\hat{v}(\hat{x})$ is convex and compact.

Proof See text. $\blacksquare$