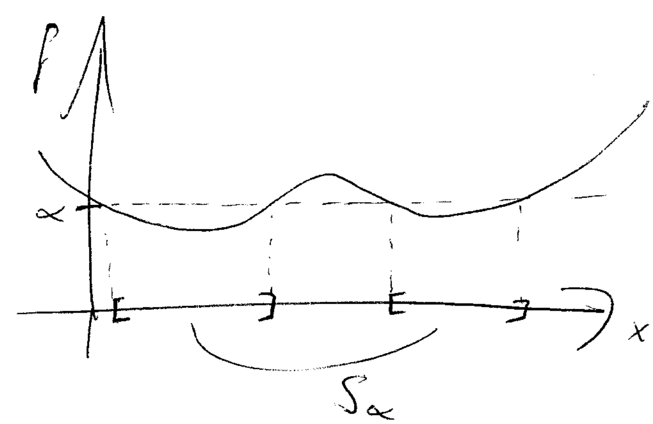
~~Local Sets~~ Level Sets.

(Already done on p 31)

$S \subseteq \mathbb{R}^n$, $f : S \to \mathbb{R}$.

Then the ~~level~~ set $S_\alpha = \{x \in S : f(x) \le \alpha\}$ is a **level set** of f.



Lemma Let $C \subseteq \mathbb{R}^n$ nonempty, convex.

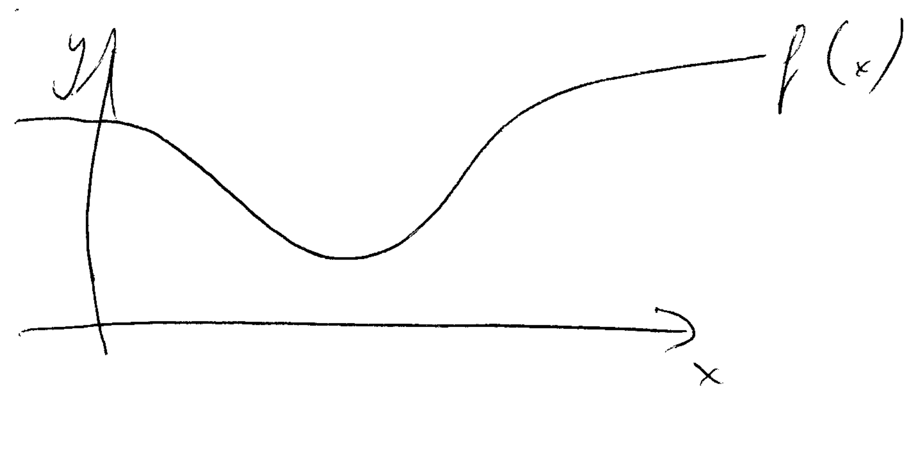
$f : C \to \mathbb{R}$ convex.

Then for any α, ~~S_α~~ the level set $C_\alpha = \{x \in C : f(x) \le \alpha\}$ is convex.

Proof Obvious.

Converse not true:

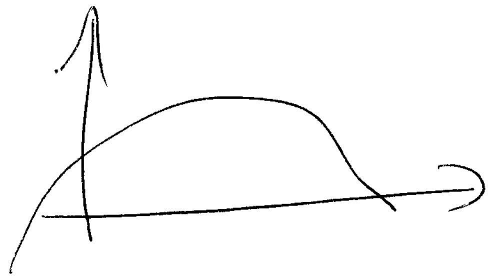
eg



Concave fns

$C \subseteq \mathbb{R}^n$, $f: C \to \mathbb{R}$.

f is called (strictly) concave if $-f$ is (strictly) convex.



~~Results for convex~~

Comparable results for concave fns as we had for convex fns,

so in particular we have a condition for maximizing concave functions.

f concave $\Leftrightarrow$ Hessian nsd

ie $x^T H x \leq 0 \quad \forall x$.

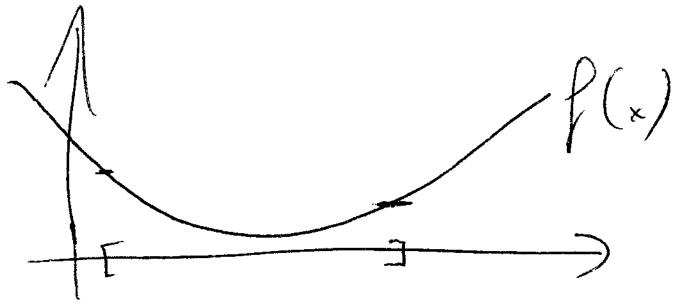
Maximizing convex functions.

Local information not so useful.

$$\max_{x \in C} f(x)$$

C convex

f convex.



May ~~cutt~~ have several local maxima.

No local information leading to global optimum.

Do have a ~~suffi~~ ~~neces~~ necessary condition for optimality:

Theorem $f : \mathbb{R}^n \to \mathbb{R}$ convex, $C \subseteq \mathbb{R}^n$ convex.

~~&~~ Consider the problem $\max_{x \in C} f(x)$.

~~Then~~ If $\bar{x} \in C$ is a local optimum solution, then

$$\xi^T(x - \bar{x}) \leq 0$$

for each $x \in S$, where ξ is any subgradient of f at $\bar{x}$.

Theorem ~~If~~ $f : \mathbb{R}^n \to \mathbb{R}$ convex, $C \subseteq \mathbb{R}^n$ convex, compact.

~~Then~~ Consider the problem $\max_{x \in C} f(x)$.

Then there is an optimal solution to this problem ~~by~~ which is in ∂C. Proof: Use Carathéodory. (Needs care: ~~cy~~ (almost getting into fixed point stuff.)