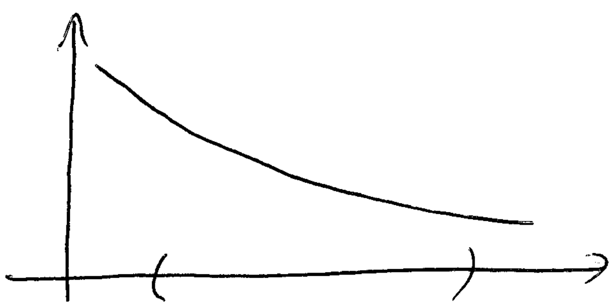
Weierstrass' Theorem

Let S be a nonempty compact set and let $f: S \to \mathbb{R}$ be continuous on S. Then the problem $\min \{f(x) : x \in S\}$ attains its minimum, that is, there exists a global minimizer to this problem.

Proof See text //.

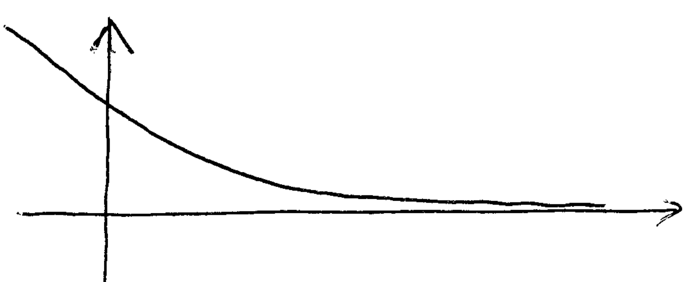
Need closed:



$$\min \ (x-3)^2 \quad \text{s.t.} \quad 1 < x < 2$$

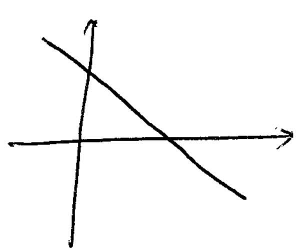
$f(x) \to 1$ as $x \to 2$

Need bounded:



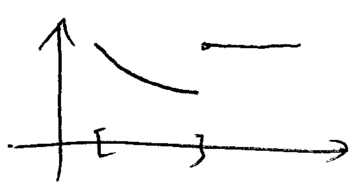
$$\min \ e^{-x}, \quad x \geq 0$$

or



$$\min \ 1-x, \quad x \geq 0.$$

Need continuous:



$$f(x) = \begin{cases} (x-3)^2 & x \leq 2 \\ 2 & x \geq 2 \end{cases}$$

$$\min_{1 \leq x \leq 2} f(x)$$