TABLE OF INTEGRALS

1. Elementary integrals

All of these follow immediately from the table of derivatives. They should be memorized.

\[ \int \sec x \, dx = \ln |\sec x + \tan x| + C \]

\[ \int c f(x) \, dx = c \int f(x) \, dx \]

\[ \int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx \]

\[ \int c \, dx = cx + C \]

\[ \int x^r \, dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1) \]

\[ \int \frac{1}{x} \, dx = \ln |x| + C \]

\[ \int e^x \, dx = e^x + C \]

\[ \int \sin x \, dx = -\cos x + C \]

\[ \int \cos x \, dx = \sin x + C \]

\[ \int \frac{1}{x^2 + 1} \, dx = \arctan x + C \]

\[ \int \frac{1}{\sqrt{1 - x^2}} \, dx = \arcsin x + C \]

\[ \int \tan x \, dx = -\ln |\cos x| + C = \ln |\sec x| + C \]

\[ \int \cot x \, dx = \ln |\sin x| + C = -\ln |\csc x| + C \]

\[ \int \sec x \, dx = \ln |\sec x + \tan x| + C \]

\[ \int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C \]
2. A SELECTION OF MORE COMPLICATED INTEGRALS

These begin with the two basic formulas, change of variables and integration by parts.

\[ \int f(g(x))g'(x) \, dx = \int f(u) \, du \text{ where } u = g(x), \, du = g'(x) \, dx \text{ (change of variables)} \]

\[ \int f(g(x)) \, dx = \int f(u) \frac{du}{du} \, du \text{ where } u = g(x) \text{ (different form of the same change of variables)} \]

\[ \int e^{cx} \, dx = \frac{1}{c} e^{cx} + C \quad (c \neq 0) \]

\[ \int a^x \, dx = \frac{1}{\ln a} a^x + C \text{ (for } a > 0, \ a \neq 1) \]

\[ \int \ln x \, dx = x \ln x - x + C \]

\[ \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \frac{x}{a} + C, \ a \neq 0 \]

\[ \int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C, \ a \neq 0 \]

\[ \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \frac{x}{a} + C, \ a > 0 \]

\[ \int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left| x + \sqrt{x^2 + a^2} \right| + C \]

To compute \( \int \frac{1}{x^2 + bx + c} \, dx \) we complete the square

\[ x^2 + bx + c = x^2 + bx + \frac{b^2}{4} + c - \frac{b^2}{4} = \left( x + \frac{b}{2} \right)^2 + c - \frac{b^2}{4} \]

If \( c - \frac{b^2}{4} > 0 \), set it equal to \( a^2 \); if \( < 0 \) equal to \( -a^2 \); and if \( = 0 \) forget it. In any event you will arrive after the change of variables \( u = x + \frac{b}{2} \) at one of the three integrals

\[ \int \frac{1}{u^2 + a^2} \, du, \quad \int \frac{1}{u^2 - a^2} \, du, \quad \int \frac{1}{u^2} \, du \]

\[ \int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \frac{1}{2} \left( x \sqrt{x^2 + a^2} + a^2 \ln \left| x + \sqrt{x^2 + a^2} \right| \right) + C \]

\[ \int x^n e^{cx} \, dx = x^n \frac{e^{cx}}{c} - \frac{n}{c} \int x^{n-1} e^{cx} \, dx, \ c \neq 0. \text{ This is to be used repeatedly until you arrive at the case } n = 0, \text{ which you can do easily.} \]