A

Name:

Math Models of Operations Research, MATP 4700/ DSES 4770.

First Exam, Tuesday, October 1, 2002.

You may use any result from your notes or a homework that is clearly stated. You may use one sheet of handwritten notes, but no other sources. The exam consists of five questions, and lasts one hundred minutes.

Solutions.

Q1		/ 20
Q2		/ 12
Q3		/ 10
Q4		/ 18
Q5		/ 40
Total		/ 100
Grade		

1. (20 points) Wiley Wiz is a mutual fund manager trying to decide how to divide up \$12 million between domestic and foreign stocks. Domestic stocks have been returning 11% per year and foreign 17%. Naturally, Wiley would like to maximize the annual return from his investments. Still, he wants to exercise some caution. No more than \$10 million of the fund should go into domestic stocks and no more than \$7 million into foreign. Also, at least half as much should be invested in domestic as foreign to maintain some balance.

   (a) (10 points) Formulate a mathematical programming model with 5 main constraints to decide Wiley's optimal investment plan using decision variables x_1 := millions of dollars invested in domestic stocks and x_2 := millions of dollars invested in foreign stocks.

   (b) (10 points) Using a 2-dimensional plot, solve your model graphically for an optimal investment plan.

See solutions to problems from text.

2. (12 points) Find a basic feasible solution with basic variables x_2 and x_4 for the linear programming problem

$$\begin{array}{lrcrcrcrcl} \text{min} & 2x_1 & + & 3x_2 & + & 2x_3 & - & 5x_4 & & \\ \text{subject to} & 2x_1 & + & x_2 & - & x_3 & - & 2x_4 & = & 2 \\ & -x_1 & + & 2x_2 & + & 2x_3 & - & 3x_4 & = & 5 \\ & \multicolumn{9}{l}{x_i \geq 0, \quad i = 1, \ldots, 4.} \end{array}$$

Is this basic feasible solution optimal?

Pivot to make x_2 basic:

$$\begin{array}{lrrrrl} \text{min} & 6 - 4x_1 & & +5x_3 & +x_4 & \\ \text{st.} & 2x_1 & +x_2 & -x_3 & -2x_4 & = 2 \\ & -5x_1 & & +4x_3 & +x_4 & = 1 \\ & \multicolumn{5}{l}{x_i \geq 0} \end{array}$$

Pivot to make x_4 basic:

$$\begin{array}{lrrrrl} \text{min} & 7 + x_1 & & +x_3 & & \\ \text{s.t.} & -8x_1 & +x_2 & +7x_3 & & = 4 \\ & -5x_1 & & +4x_3 & +x_4 & = 1 \\ & \multicolumn{5}{l}{x_i \geq 0} \end{array}$$

BFS: $x_2 = 4$, $x_4 = 1$, x_1, x_3 nonbasic.

Value: 7

Yes, optimal.

3. (10 points)

   (a) (7 points) Put the following problem into standard form:

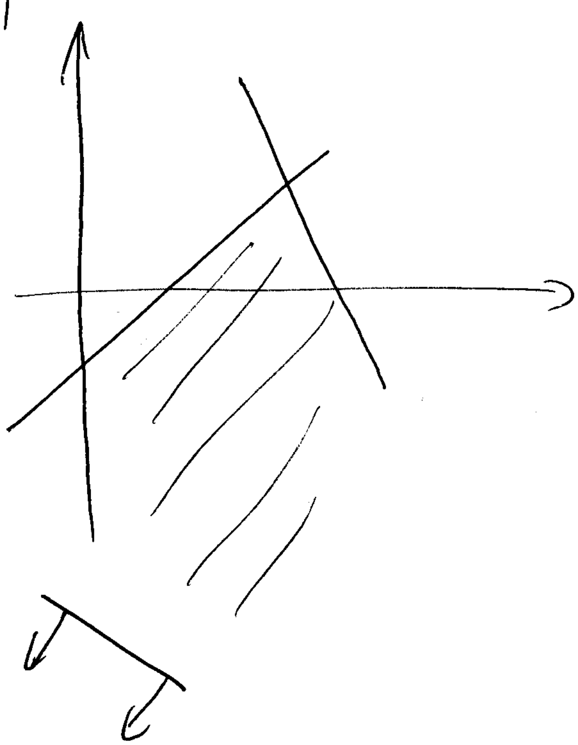
$$\begin{array}{lrcrcl} \text{min} & x_1 & + & x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \geq & 1 \\ & x_1 & + & 3x_2 & \leq & 3 \\ & \multicolumn{5}{c}{x_1 \geq 0,\ x_2 \text{ free}} \end{array}$$

   (b) (3 points, no partial credit. Lose one point for incorrect answer.) For the problem in part (a), use the graphical method with the original variables x_1 and x_2 to determine which of the following is true:

   - **A:** The LP has an optimal solution.
   - **B:** The LP is infeasible.
   - **C:** The LP has an unbounded optimal value.

   Circle your choice: **A** **B** **C**

(b)



(a)

$$\begin{array}{llrrrrrl} \text{min} & x_1 & +x_2 & -x_3 & & & & \\ \text{s.t.} & x_1 & -x_2 & +x_3 & -x_4 & & = & 1 \\ & x_1 & +3x_2 & -3x_3 & & +x_5 & = & 3 \end{array}$$

all $x_i \geq 0$.

4. (18 points; 3 points for each correct answer, lose one point for each incorrect answer. No partial credit.) For each part, either the statement is always true or there exists a counterexample to the statement. If the statement is always true, circle **TRUE**, otherwise circle **FALSE**.

   (a) If an artificial variable remains as a basic variable after the artificial problem has been solved, then the original linear program is infeasible.

   TRUE FALSE

   (b) Given a linear program in canonical form, if we pivot using the simplex rule, then a strict decrease in the objective function value is obtained.

   TRUE FALSE

   (c) A linear program in canonical form always has a feasible solution.

   TRUE FALSE

   (d) If the feasible region for a linear program is unbounded, then no finite optimal value exists.

   TRUE FALSE

   (e) Every tableau in standard form can be put into canonical form by an appropriate sequence of pivots.

   TRUE FALSE

   (f) If a linear program has a unique optimal solution, that optimum most occur at an extreme point of the feasible region.

   TRUE FALSE

5. (40 points; each part is worth 5 points.) Consider the linear programming problem with tableau:

$$M_0 = \begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline -3 & 0 & 2 & 0 & 0 & -1 \\ \hline 0 & 0 & -2 & 1 & 0 & 1 \\ 3 & 1 & 2 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 & -1 \end{array}$$

The corresponding basic feasible solution is $\bar{x} = (3, 0, 0, 1, 0)$, and x_1, x_3, and x_4 are the basic variables.

(a) Construct a graph of the feasible set in x_2x_5 space and label each hyperplane as $x_i = 0$ for some i, $i = 1, \ldots, 5$.

(b) The given basic feasible solution is degenerate. What feature of the graph corresponds to this property?

(c) This tableau is not in optimal form. If you try to take a simplex pivot, what is the simplex direction?

(d) What is the minimum ratio for the simplex direction in part (c)?

(e) What will be the new point after the pivot? Why? (Don't make the pivot. It does not suffice to quote from part (f).)

(f) After making the pivot, the following tableau is obtained:

$$M_1 = \begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline -3 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & -2 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 & 0 \end{array}$$

Find a point x^* and a vector v such that the set of optimal solutions is $x^* + tv$ for any scalar $t \geq 0$.

(g) Graph the set of optimal solutions in your picture in part (a).

(h) Plot a contour of the objective function in your picture in part (a).

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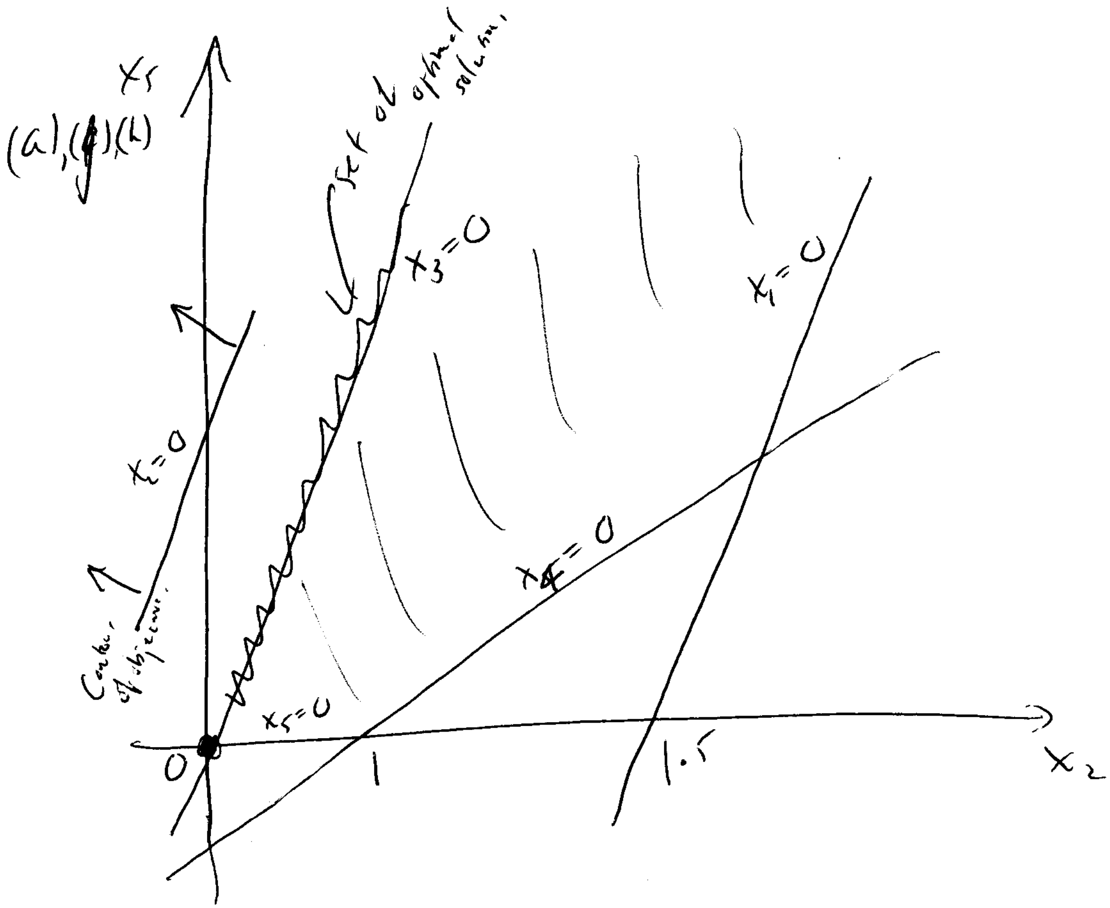

(b) Three lines intersect at the origin

(c) Increase x_5. $\quad \Delta x_5 = (1, 0, -1, +1, 1)$

(d) Minimum ratio: $\quad 0/1 = 0$.

(e) $x = \bar{x} + \lambda \Delta x_5 = \bar{x} + 0 \Delta x_5 = \bar{x}$.

(f) Look at x_2 column: $\quad v = \Delta x_2 = (0, 1, 0, 1, 2)$

$x^* = \bar{x} = (3, 0, 0, 1, 0)$.