

# Chapter 9

## Exercise Solutions

**9-1. (a)** Nodes: 1,2,3,4,5; arcs (directed): (3,2), (3,4), (3,5); edges (undirected): (1,2), (1,3), (1,4), (2,5), (4,5) **(b)** Sequence 1 - 3 - 4 - 5 is a path because it is a nonrepeating, connected sequence that conforms to direction on arcs. Sequence 2 - 5 - 3 - 4 is not a path because it violated direction on arc (3,5). Sequence 1 - 3 - 2 - 5 - 4 is a path because it is a nonrepeating, connected sequence that conforms to direction on arcs. Sequence 1 - 3 - 4 - 1 - 2 is not a path because it repeats node 1. **(c)** Replace each edge with 2 opposed arcs of the same length as the edge. Arcs of the original graph are unchanged.

**9-2. (a)** Nodes: 1,2,3,4,5; arcs (directed): (2,4), (2,5), (3,5); edges (undirected): (1,2), (1,3), (2,3), (4,5) **(b)** Sequence 3 - 2 - 1 - 3 - 5 is not a path because it repeats node 3. Sequence 3 - 2 - 8 is a path because it is a nonrepeating, connected sequence that conforms to direction on arcs. Sequence 1 - 3 - 2 - 4 - 5 is a path because it is a nonrepeating, connected sequence that conforms to direction on arcs. Sequence 1 - 3 - 5 - 2 is not a path because it violates direction on arc (2,5). **(c)** replace each edge with 2 opposed arcs of the same length as the edge

**9-3. (a)** Paths from 1 to 2 are 1 - 2 of length 10, 1 - 3 - 2 of length 8, 1 - 4 - 5 - 2 of length 13, 1 - 3 - 4 - 5 - 2 of length 18, and 1 - 3 - 5 - 2 of length 6. The last is shortest. Similarly, 1 - 3, length 3 for node 3, 1 - 4, length 7 for node 4, 1 - 3 - 5, length 4 for node 5. **(b)** In the shortest path 1 - 3 - 5 - 2 from 1 to 2, subpaths are 1 - 3, which is the optimal path for 3, 1 - 3 - 5, which is the optimal path for 5, 3 - 5, which is the shortest path from 3 to 5, 3 - 5 - 2, which is

the shortest path from 3 to 2, and 5 - 2, which is the shortest path from 5 to 2. **(c)**  $\nu[1] = 0$ ,  $\nu[2] = 6$ ,  $x_{1,3}[2] = x_{3,5}[2] = x_{2,5}[2] = 1$ ,  $\nu[3] = 3$ ,  $x_{1,3}[3] = 1$ ,  $\nu[4] = 7$ ,  $x_{1,4}[4] = 1$ ,  $\nu[5] = 4$ ,  $x_{1,3}[5] = x_{3,5}[5] = 1$  **(d)**  $\nu[1] = 0$ ,  $\nu[2] = \min\{\nu[1] + 10, \nu[3] + 5, \nu[5] + 2\}$ ,  $\nu[3] = \min\{\nu[1] + 3\}$ ,  $\nu[4] = \min\{\nu[1] + 7, \nu[3] + 9, \nu[5] + 4\}$ ,  $\nu[5] = \min\{\nu[2] + 2, \nu[3] + 1, \nu[4] + 4\}$  **(e)** Substitute directly in the functional equations. **(f)** Functional equations are sufficient in the absence of negative dicycles. Here, positive arc/edge lengths preclude negative dicycles.

**9-4. (a)** Paths from 1 to 2 are 1 - 3 - 2, length 11, and shortest 1 - 2, length 2. Similarly, 1 - 2 - 3, length 5 for node 3, 1 - 2 - 5 - 4, length 8 for node 4, 1 - 2 - 5, length 7 for node 5. **(b)** In the shortest path 1 - 2 - 3 from 1 to 3, subpaths are 1 - 2, which is shortest for 1 to 2, and 2 - 3 which is the shortest path from 2 to 3. **(c)**  $\nu[1] = 0$ ,  $\nu[2] = 2$ ,  $x_{1,2}[2] = 1$ ,  $\nu[3] = 5$ ,  $x_{1,2}[3] = x_{2,3}[3] = 1$ ,  $\nu[4] = 8$ ,  $x_{1,2}[4] = x_{2,5}[4] = x_{4,5}[4] = 1$ ,  $\nu[5] = 7$ ,  $x_{1,2}[5] = x_{2,5}[5] = 1$  **(d)**  $\nu[1] = 0$ ,  $\nu[2] = \min\{\nu[1] + 2, \nu[3] + 3\}$ ,  $\nu[3] = \min\{\nu[1] + 8, \nu[2] + 3\}$ ,  $\nu[4] = \min\{\nu[2] + 8, \nu[5] + 1\}$ ,  $\nu[5] = \min\{\nu[2] + 5, \nu[3] + 6, \nu[4] + 1\}$  **(e)** Substitute directly in the functional equations. **(f)** Functional equations are sufficient in the absence of negative dicycles. Positive arc/edge lengths preclude negative dicycles.

**9-5. (a)** 1 - 3 - 2, length 4 for 1 to 2, 1 - 3, length 3 for 1 to 3, 1 - 3 - 2 - 4, length 10 for 1 to 4, 2 - 1, length 8 for 2 to 1, 2 - 4 - 3, length 10 for 2 to 3, 2 - 4, length 6 for 2 to 4, 3 - 1, length 3 for 3 to 1, 3 - 2, length 1 for 3 to 2, 3 - 2 - 4, length 7 for 3 to 4, 4 - 3 - 1, length 7 for 4 to 1, 4 - 3 - 2, length 5 for 4 to 2, 4 - 3 length 3 for 4

to 3. (b) Shortest 1 to 4 path 1-3-2-4 has subpaths 1-3, 1-3-2, 3-2, 3-2-4, and 2-4. All are shortest for their respective endpoints in part (a). (c)  $\nu[k][k] = 0, k = 1, \dots, 4, \nu[1][2] = 4, x_{1,3}[1][2] = x_{3,2}[1][2] = 1, \nu[1][3] = 3, x_{1,3}[1][3] = 1, \nu[1][4] = 10, x_{1,3}[1][4] = x_{3,2}[1][4] = x_{2,4}[1][4] = 1, \nu[2][1] = 8, x_{1,2}[2][1] = 1, \nu[2][3] = 10, x_{2,4}[2][3] = x_{4,3}[2][3] = 1, \nu[2][4] = 6, x_{2,4}[2][4] = 1, \nu[3][1] = 3, x_{1,3}[3][1] = 1, \nu[3][2] = 1, x_{3,2}[3][2] = 1, \nu[3][4] = 7, x_{3,2}[3][4] = x_{2,4}[3][4] = 1, \nu[4][1] = 7, x_{4,3}[4][1] = x_{1,3}[4][1] = 1, \nu[4][2] = 5, x_{4,3}[4][2] = x_{3,2}[4][2] = 1, \nu[4][3] = 3, x_{4,3}[4][3] = 1$  (d)  $\nu[k][k] = 0, k = 1, \dots, 4, \nu[1][2] = \min\{8, \nu[1][3] + \nu[3][2], \nu[1][4] + \nu[4][1]\}, \nu[1][3] = \min\{3, \nu[1][2] + \nu[2][3], \nu[1][4] + \nu[4][2]\}, \nu[1][4] = \min\{\nu[1][2] + \nu[2][4], \nu[1][3] + \nu[3][4]\}, \nu[2][1] = \min\{8, \nu[2][3] + \nu[3][1], \nu[2][4] + \nu[4][1]\}, \nu[2][3] = \min\{\nu[2][1] + \nu[1][3], \nu[2][4] + \nu[4][3]\}, \nu[2][4] = \min\{6, \nu[2][1] + \nu[1][4], \nu[2][3] + \nu[3][4]\}, \nu[3][1] = \min\{3, \nu[3][2] + \nu[2][1], \nu[3][4] + \nu[4][1]\}, \nu[3][2] = \min\{1, \nu[3][1] + \nu[1][2], \nu[3][4] + \nu[4][2]\}, \nu[3][4] = \min\{\nu[3][1] + \nu[1][4], \nu[3][2] + \nu[2][4]\}, \nu[4][1] = \min\{\nu[4][2] + \nu[2][1], \nu[4][3] + \nu[3][1]\}, \nu[4][2] = \min\{6, \nu[4][1] + \nu[1][2], \nu[4][3] + \nu[3][2]\}, \nu[4][3] = \min\{4, \nu[4][1] + \nu[1][3], \nu[4][2] + \nu[2][3]\}$  (e) Substitute directly in the functional equations. (f) Functional equations are sufficient in the absence of negative dicycles. Positive arc/edge lengths preclude negative dicycles.

9-6. (a) 1-2, length 5 for 1 to 2, 1-3, length 3 for 1 to 3, 1-3-4, length 5 for 1 to 4, 2-1, length 5 for 2 to 1, 2-1-3, length 8 for 2 to 3, 2-4, length 1 for 2 to 4, 3-1, length 3 for 3 to 1, 3-4-2, length 3 for 3 to 2, 3-4, length 2 for 3 to 4, 4-2-1, length 6 for 4 to 1, 4-2, length 1 for 4 to 2, 4-2-1-3, length 9 for 4 to 3. (b) Shortest 4 to 2 path 4-2-1-3 has subpaths 4-2, 4-2-1, 2-1, 2-1-3 and 1-3. (c)  $\nu[k][k] = 0, k = 1, \dots, 4, \nu[1][2] = 5, x_{1,2}[1][2] = 1, \nu[1][3] = 3, x_{1,3}[1][3] = 1, \nu[1][4] = 5, x_{1,3}[1][4] = x_{3,4}[1][4] = 1, \nu[2][1] = 5, x_{1,2}[2][1] = 1, \nu[2][3] = 8, x_{1,2}[2][3] = x_{1,3}[2][3] = 1, \nu[2][4] = 1, x_{2,4}[2][4] = 1, \nu[3][1] = 3, x_{1,3}[3][1] = 1, \nu[3][2] = 3, x_{3,4}[3][2] = x_{2,4}[3][2] = 1, \nu[3][4] = 2, x_{3,4}[3][4] = 1, \nu[4][1] = 6, x_{2,4}[4][1] = x_{1,2}[4][1] = 1, \nu[4][2] = 1, x_{2,4}[4][2] = 1, \nu[4][3] = 9,$

$x_{2,4}[4][3] = x_{1,2}[4][3] = x_{1,3}[4][3] = 1$  (d)  $\nu[k][k] = 0, k = 1, \dots, 4, \nu[1][2] = \min\{5, \nu[1][3] + \nu[3][2], \nu[1][4] + \nu[4][1]\}, \nu[1][3] = \min\{3, \nu[1][2] + \nu[2][3], \nu[1][4] + \nu[4][2]\}, \nu[1][4] = \min\{6, \nu[1][2] + \nu[2][4], \nu[1][3] + \nu[3][4]\}, \nu[2][1] = \min\{5, \nu[2][3] + \nu[3][1], \nu[2][4] + \nu[4][1]\}, \nu[2][3] = \min\{\nu[2][1] + \nu[1][3], \nu[2][4] + \nu[4][3]\}, \nu[2][4] = \min\{1, \nu[2][1] + \nu[1][4], \nu[2][3] + \nu[3][4]\}, \nu[3][1] = \min\{3, \nu[3][2] + \nu[2][1], \nu[3][4] + \nu[4][1]\}, \nu[3][2] = \min\{\nu[3][1] + \nu[1][2], \nu[3][4] + \nu[4][2]\}, \nu[3][4] = \min\{2, \nu[3][1] + \nu[1][4], \nu[3][2] + \nu[2][4]\}, \nu[4][1] = \min\{\nu[4][2] + \nu[2][1], \nu[4][3] + \nu[3][1]\}, \nu[4][2] = \min\{1, \nu[4][1] + \nu[1][2], \nu[4][3] + \nu[3][2]\}, \nu[4][3] = \min\{\nu[4][1] + \nu[1][3], \nu[4][2] + \nu[2][3]\}$  (e) Substitute directly in the functional equations. (f) Functional equations are sufficient in the absence of negative dicycles. Here, positive arc/edge lengths preclude negative dicycles.

9-7. (a) 1-2, length -5 for 1 to 2, 1-3, length 10 for 1 to 3, 1-3-4, length 12 for 1 to 4. (b) 1-2-4-1, 1-3-4-1 (c) Not negative because  $(-5) + (40) + (-20) = 15 \geq 0$ . Negative dicycle because  $(10) + (2) + (-20) = -8 < 0$ . (d) With negative dicycles in the problem, functional equations are no longer sufficient. Subpaths of optimal paths may not be optimal.

9-8. (a) 1-4-2, length 10 for 1 to 2, 1-3, length 30 for 1 to 3, 1-4, length 10 for 1 to 4. (b) 1-3-4-2-1, 1-4-2-1 (c) Not negative because  $(30) + (-16) + (0) + (-12) = 2 \geq 0$ . Negative dicycle because  $(10) + (0) + (-12) = -2 < 0$ . (d) With negative dicycles in the problem, functional equations are no longer sufficient. Subpaths of optimal paths may not be optimal.

9-9. (a) This is a one node to all others problem with no negative dicycles.

(b)

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$
$t = 0$	0	$\infty$	$\infty$	$\infty$	$\infty$
$t = 1$		10	3	7	
$t = 2$		8			4
$t = 3$		6			
$t = 4$					

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$
$t = 1$		1	1	1	
$t = 2$		3			3
$t = 3$		5			
$t = 4$					

(c) For the shortest 1 to 2 path,  $d[2] = 5$ ,  $d[5] = 3$ ,  $d[3] = 1$  giving path 1 - 3 - 5 - 2. Similar backtracking from the destination node recovers the rest of the optimal paths in Exercise 9-3(a).

(d) Paths of 1 or 2 arcs/edges from node 1 to node 2 are 1 - 2, length 10, and shortest 1 - 3 - 2, length 8, which is the  $\nu[2]$  value after  $t = 2$  in (b). Corresponding, at most 2 steps optimal paths for other nodes are 1 - 3, length 3 for 1 to 3, 1 - 4, length 7 for 1 to 4, 1 - 3 - 5, length 4 for 1 to 5. (e) The number of nodes is 5, so no path can have more than  $5 - 1 = 4$  arcs/edges. Thus, with no negative dicycles, labels will be correct after  $t = 4$ .

9-10. (a) This is a one node to all others problem with no negative dicycles.

(b)

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$
$t = 0$	0	$\infty$	$\infty$	$\infty$	$\infty$
$t = 1$		2	8		
$t = 2$			5	10	7
$t = 3$				8	
$t = 4$					

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$
$t = 1$		1	1		
$t = 2$			2	2	2
$t = 3$				5	
$t = 4$					

(c) For the shortest 1 to 4 path,  $d[4] = 5$ ,  $d[5] = 2$ ,  $d[2] = 1$ , giving shortest path 1 - 2 - 5 - 4. Similar backtracking from the destination node recovers the rest of the optimal paths in Exercise 9-4(a). (d) Paths for node 1 to node 2 in 2 or fewer arcs/edges are 1 - 3 - 2 length 11, and shortest 1 - 2, length 2, which agrees with the  $\nu[2]$  value after  $t = 2$  in (b). Corresponding, at most 2 steps optimal paths for other nodes are 1 - 2 - 3, length 5 for 1 to 3, 1 - 2 - 4, length 10 for 1 to 4, 1 - 2 - 5, length 7 for 1 to 5. (e) The

number of nodes is 5, so no path can have more than  $5 - 1 = 4$  arcs/edges. Thus, with no negative dicycles, labels will be correct after  $t = 4$ .

9-11. (a)

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$
$t = 0$	0	$\infty$	$\infty$	$\infty$
$t = 1$		-5	10	
$t = 2$				12
$t = 3$	-8			
$t = 4$		-13	2	

	$d[1]$	$d[2]$	$d[3]$	$d[4]$
$t = 1$		1	1	
$t = 2$				3
$t = 3$	4			
$t = 4$				

There is a negative dicycle because  $\nu[2]$  changed at iteration  $t = 4 =$  the number of nodes in this graph. Backtracking from 2,  $d[2] = 1$ ,  $d[1] = 4$ ,  $d[4] = 3$ ,  $d[3] = 1$ , recovers negative dicycle 1 - 3 - 4 - 1

(b)

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$
$t = 0$	0	$\infty$	$\infty$	$\infty$
$t = 1$			30	10
$t = 2$		10		
$t = 3$	-2			
$t = 4$			28	8

	$d[1]$	$d[2]$	$d[3]$	$d[4]$
$t = 1$			1	1
$t = 2$		4		
$t = 3$	2			
$t = 4$				

There is a negative dicycle because  $\nu[3]$  changed at iteration  $t = 4 =$  the number of nodes in this graph. Backtracking,  $d[3] = 1$ ,  $d[1] = 2$ ,  $d[2] = 4$ ,  $d[4] = 1$  recovers negative dicycle 1 - 4 - 2 - 1

9-12. (a) This is an all nodes to all others problem with no negative dicycles.

(b)

$\nu^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	8	3	$\infty$
$i = 2$	8	0	$\infty$	6
$i = 3$	3	1	0	$\infty$
$i = 4$	$\infty$	$\infty$	4	0

$\nu^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	8	3	$\infty$
$i = 2$	8	0	<b>11</b>	6
$i = 3$	3	1	0	$\infty$
$i = 4$	$\infty$	$\infty$	4	0

$\nu^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	8	3	<b>14</b>
$i = 2$	8	0	11	6
$i = 3$	3	1	0	<b>7</b>
$i = 4$	$\infty$	$\infty$	4	0

$\nu^{(3)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	<b>4</b>	3	<b>10</b>
$i = 2$	8	0	11	6
$i = 3$	3	1	0	7
$i = 4$	<b>7</b>	<b>5</b>	4	0

$\nu^{(4)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	4	3	10
$i = 2$	8	0	<b>10</b>	6
$i = 3$	3	1	0	7
$i = 4$	7	5	4	0

$d^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	
$i = 2$	2			2
$i = 3$	3	3		
$i = 4$			4	

$d^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	
$i = 2$	2		<b>1</b>	2
$i = 3$	3	3		
$i = 4$			4	

$d^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	<b>2</b>
$i = 2$	2		1	2
$i = 3$	3	3		<b>2</b>
$i = 4$			4	

$d^{(3)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$			<b>3</b>	1
$i = 2$	2		1	2
$i = 3$	3	3		2
$i = 4$	<b>3</b>	<b>3</b>	4	

$d^{(4)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		3	1	2
$i = 2$	2		<b>4</b>	2
$i = 3$	3	3		2
$i = 4$	3	3	4	

(c) For the optimal path from node 1 to node 4 we backtrack  $d[1][4] = 2$ ,  $d[1, 2] = 3$ ,  $d[1, 3] = 1$  to obtain optimal path 1 - 3 - 2 - 4. Similar backtracking from the destination node recovers other optimal paths of Exercise 9-5(a).  
 (d) Shortest paths with intermediates at most 2 are 1 - 2, length 8, 1 - 3, length 3, 1 - 2 - 4, length 14, 2 - 1, length 8, 2 - 1 - 3, length 11, 2 - 4, length 6, 3 - 1, length 3, 3 - 2, length 1, 3 - 2 - 4, length 7, no path 4 to 1, no path 4 to 2, 4 - 3 length 4. All these match values in the  $\nu^{(2)}$  table of part (b).

9-13. (a) This is an all nodes to all others problem with no negative dicycles

(b)

$\nu^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	5	3	6
$i = 2$	5	0	$\infty$	1
$i = 3$	3	$\infty$	0	2
$i = 4$	$\infty$	1	$\infty$	0

$\nu^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	5	3	6
$i = 2$	5	0	<b>8</b>	1
$i = 3$	3	<b>8</b>	0	2
$i = 4$	$\infty$	1	$\infty$	0

$\nu^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	5	3	6
$i = 2$	5	0	8	1
$i = 3$	3	8	0	2
$i = 4$	<b>6</b>	1	<b>9</b>	0

$\nu^{(3)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	5	3	5
$i = 2$	5	0	8	1
$i = 3$	3	8	0	2
$i = 4$	6	1	9	0

$\nu^{(4)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	5	3	5
$i = 2$	5	0	8	1
$i = 3$	3	3	0	2
$i = 4$	6	1	9	0

$d^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	1
$i = 2$	2			2
$i = 3$	3			3
$i = 4$		4		

$d^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	1
$i = 2$	2		1	2
$i = 3$	3	1		3
$i = 4$		4		

$d^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	1
$i = 2$	2		1	2
$i = 3$	3	1		3
$i = 4$	2	4	1	

$d^{(3)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	3
$i = 2$	2		1	2
$i = 3$	3	1		3
$i = 4$	2	4	1	

$d^{(4)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	3
$i = 2$	2		1	2
$i = 3$	3	4		3
$i = 4$	2	4	1	

(c) For the shortest node 4 to node 3 path we backtrack  $d[4][3] = 1$ ,  $d[4][1] = 2$ ,  $d[4][2] = 4$  to recover shortest path 4 - 2 - 1 - 3. Similar backtracking from the destination node recovers other optimal paths of Exercise 9-6(a).

(d) Shortest paths with intermediates at most 2 are 1-2, length 5, 1-3, length 3, 1-4, length 6, 2-1, length 5, 2-1-3, length 8, 2-4, length 1, 3-1, length 3, 3-1-2, length 8, 3-4, length 2, 4-2-1, length 6, 4-2, length 1, 4-2-1-3, length 9. All these agree with values in the  $\nu^{(2)}$  table of part (b).

9-14. (a)

$\nu^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	-5	10	$\infty$
$i = 2$	$\infty$	0	$\infty$	40
$i = 3$	$\infty$	$\infty$	0	2
$i = 4$	-20	$\infty$	$\infty$	0

$\nu^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	-5	10	$\infty$
$i = 2$	$\infty$	0	$\infty$	40
$i = 3$	$\infty$	$\infty$	0	2
$i = 4$	-20	-25	-10	0

$\nu^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	-5	10	35
$i = 2$	$\infty$	0	$\infty$	40
$i = 3$	$\infty$	$\infty$	0	2
$i = 4$	-20	-25	-10	0

$\nu^{(3)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	-5	10	12
$i = 2$	$\infty$	0	$\infty$	40
$i = 3$	$\infty$	$\infty$	0	2
$i = 4$	-20	-25	-10	-8

$d^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	
$i = 2$				2
$i = 3$				3
$i = 4$	4			

$d^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	
$i = 2$				2
$i = 3$				3
$i = 4$	4	1	1	

$d^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		1	1	2
$i = 2$				2
$i = 3$				3
$i = 4$	4	1	1	

$d^{(3)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$				3
$i = 2$		1	1	2
$i = 3$				3
$i = 4$	4	1	1	3

There is a negative dicycle because  $\nu[4][4] < 0$ . Backtracking  $d[4][4] = 3$ ,  $d[4][3] = 1$ ,  $d[4][1] = 4$ , which recovers negative dicycle  $4 - 1 - 3 - 4$ .

(b)

$\nu^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	$\infty$	30	10
$i = 2$	-12	0	$\infty$	$\infty$
$i = 3$	$\infty$	$\infty$	0	-16
$i = 4$	$\infty$	0	$\infty$	0

$\nu^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	$\infty$	30	10
$i = 2$	-12	0	18	-2
$i = 3$	$\infty$	$\infty$	0	-16
$i = 4$	$\infty$	0	$\infty$	0

$\nu^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	$\infty$	30	10
$i = 2$	-12	0	18	-2
$i = 3$	$\infty$	$\infty$	0	-16
$i = 4$	-12	0	18	-2

$d^{(0)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$			1	1
$i = 2$	2			
$i = 3$				3
$i = 4$		4		

$d^{(1)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$			1	1
$i = 2$	2		1	1
$i = 3$				3
$i = 4$		4		

$d^{(2)}[i, j]$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$			1	1
$i = 2$	2		1	1
$i = 3$				3
$i = 4$	2	4	1	1

There is a negative dicycle because  $\nu[4][4] < 0$ . Backtracking  $d[4][4] = 1$ ,  $d[4][1] = 2$ ,  $d[4][2] = 4$  recovers  $4 - 2 - 1 - 4$ .

9-15. (a) This is a one node to all others problem with nonnegative costs.

(b)

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$
(init)	0	$\infty$	$\infty$	$\infty$	$\infty$
$p = 1$	*	10	3	7	
$p = 3$		8	*		4
$p = 5$		6			*
$p = 2$		*			
$p = 4$				*	
(final)	0	6	3	7	4

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$
$p = 1$		1	1	1	
$p = 3$		3			3
$p = 5$		5			
$p = 2$					
$p = 4$					
(final)	-	5	1	1	3

(c) See Exercise 9-3(a). (d) Shortest paths using only the first two permanent nodes 1 and 3 are  $1 - 3 - 2$ , length 8,  $1 - 3$ , length 3,  $1 - 4$ , length 7,  $1 - 3 - 5$ , length 4. These values match those of the  $p = 3$  line of tables in part (b).

9-16. (a) This is a one node to all others problem with nonnegative costs.

(b)

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$
(init)	0	$\infty$	$\infty$	$\infty$	$\infty$
$p = 1$	*	2	8		
$p = 2$		*	5	10	7
$p = 3$			*		
$p = 5$				8	*
$p = 4$				*	
(final)	0	2	5	8	7

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$
$p = 1$		1	1		
$p = 2$			2	2	2
$p = 3$					
$p = 5$				5	
$p = 4$					
(final)	-	1	2	5	2

(c) See Exercise 9-4(a). (d) Shortest paths using only the first two permanent nodes 1 and 2 are 1-2, length 2, 1-2-3, length 5, 1-2-4, length 10, 1-2-5, length 7. These values match those of the  $p = 2$  line of tables in part (b).

9-17. (a) Bellman-Ford, Floyd-Warshall, or Dijkstra could be applied, but Dijkstra would be the best because costs are nonnegative.

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$
(init)	0	$\infty$	$\infty$	$\infty$
$p = 1$	*	7	2	
$p = 3$		5	*	
$p = 2$		*		
$p = 4$				*
(final)	0	5	2	$\infty$

	$d[1]$	$d[2]$	$d[3]$	$d[4]$
$p = 1$		1	1	
$p = 3$		3		
$p = 2$				
$p = 4$				
(final)	-	3	1	-

Optimal paths are 1-3-2, length 5, 1-3, length 2, no path to 4

(b) Bellman-Ford, Floyd-Warshall, or Dijkstra could be applied, but Dijkstra would be the best because costs are nonnegative.

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$
(init)	0	$\infty$	$\infty$	$\infty$
$p = 1$	*	18	10	
$p = 3$			*	19
$p = 2$		*		
$p = 4$				*
(final)	0	18	10	19

	$d[1]$	$d[2]$	$d[3]$	$d[4]$
$p = 1$		1	1	
$p = 3$				3
$p = 2$				
$p = 4$				
(final)	-	1	1	3

Optimal paths are 1-2, length 18, 1-3, length 10, 1-3-4, length 19.

(c) None of the algorithms are applicable because this example has a negative dicycle 2-3-4-2.

(d) Only Bellman-Ford and Floyd-Warshall could be applied because there are negative costs but no negative dicycles. For a one-to-all case, Bellman-Ford would be preferred because it requires less array storage.

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$
$t = 0$	0	$\infty$	$\infty$	$\infty$
$t = 1$		0	8	
$t = 2$			-6	22
$t = 3$				8
$t = 4$				

	$d[1]$	$d[2]$	$d[3]$	$d[4]$
$t = 1$		1	1	
$t = 2$			2	3
$t = 3$				
$t = 4$				

Optimal paths are 1-2, length 0, 1-2-3, length -6, 1-2-3-4, length 8.

9-18. (a) The digraph is acyclic. Depth first search to produce a numbering could start at  $a$ . Then  $b, c, e = 6, c, f = 5, c = 4, b = 3, a, d = 2, a = 1$ . (b) This digraph is not acyclic because it contains dicycle  $b-c-e-b$ . (c) This digraph is not acyclic because it contains dicycle  $a-d-e-c-b-a$ . (d) This digraph is acyclic. Depth first search to produce a numbering could start at  $e$ . Then  $d, a = 6, d = 5, e, c = 4, e, b = 3, e, f = 2, e = 1$ .

9-19. (a) The digraph is acyclic because every arc goes from a lower- to a higher-numbered node. (b) Taking nodes in number sequence,  $\nu[1] = 0, d[1] = -, \nu[2] = \infty, d[2] = -, \nu[3] = 2, d[3] = 1, \nu[4] = 12, d[4] = 3, \nu[5] = -1, d[5] = 3, \nu[6] = 11, d[6] = 5$ . (c) For the shortest 1 to 6 path,  $d[6] = 5, d[5] = 3, d[3] = 1$ , giving shortest path 1-3-5-6. Similar backtracking from the destination node recovers other optimal paths 1-3, 1-3-4, 1-3-5.

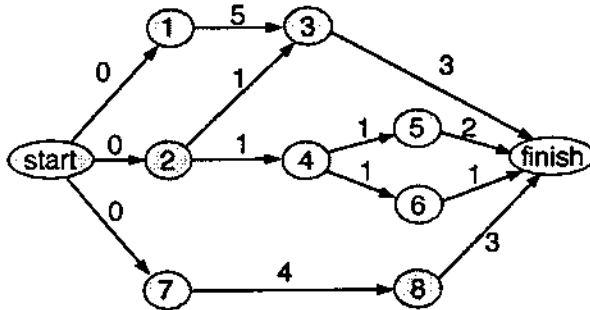
9-20. (a) The digraph is acyclic because every arc goes from a lower- to a higher-numbered

node. (b) Taking nodes in number sequence,  $\nu[1] = 0$ ,  $d[1] = -$ ,  $\nu[2] = \infty$ ,  $d[2] = -$ ,  $\nu[3] = 6$ ,  $d[3] = 1$ ,  $\nu[4] = 2$ ,  $d[4] = 1$ ,  $\nu[5] = 18$ ,  $d[5] = 4$ ,  $\nu[6] = 29$ ,  $d[6] = 5$ . (c) For the shortest 1 to 5 path,  $d[5] = 4$ ,  $d[4] = 1$ , giving shortest path 1-4-5. Similar backtracking from the destination node recovers other optimal paths 1-3, 1-4, 1-4-5-6.

9-21. (a) Not applicable because the digraph contains dicycle 2-3-2. (b) Applicable because the digraph is acyclic. The acyclic algorithm is the most efficient available when it applies. (c) Not applicable because the digraph contains dicycle 2-3-2. (d) Not applicable because the digraph contains dicycle 2-3-2.

9-22.

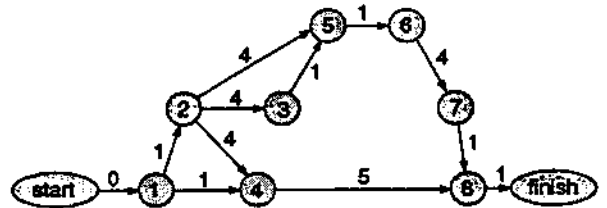
(a)



(b) Check that all arcs  $(i, j)$  have  $i < j$ . (c) Proceeding in node number sequence with  $\nu[start] = 0$  gives  $\nu[1] = 0$ ,  $d[1] = start$ ,  $\nu[2] = 0$ ,  $d[2] = start$ ,  $\nu[3] = 5$ ,  $d[3] = 1$ ,  $\nu[4] = 1$ ,  $d[4] = 2$ ,  $\nu[5] = 2$ ,  $d[5] = 4$ ,  $\nu[6] = 2$ ,  $d[6] = 4$ ,  $\nu[7] = 0$ ,  $d[7] = start$ ,  $\nu[8] = 4$ ,  $d[8] = d[7]$ ,  $\nu[finish] = 8$ ,  $d[finish] = 3$ . (d) Backtracking,  $d[finish] = 8$ ,  $d[3] = 1$ ,  $d[1] = start$  to give critical path  $start-1-3-finish$ . (e) Taking nodes in reverse number sequence with value at  $finish$  of 10 gives late starts: 1 = 2, 2 = 6, 3 = 7, 4 = 7, 5 = 8, 6 = 9, 7 = 3, 8 = 7. Then subtracting late start time minus early start time produces slacks: 1 = 2, 2 = 6, 3 = 2, 4 = 6, 5 = 6, 6 = 7, 7 = 3, 8 = 3.

9-23.

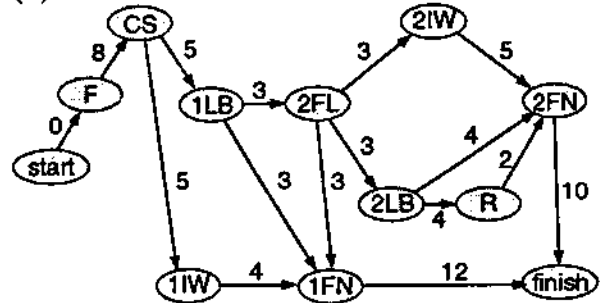
(a)



(b) Check that all arcs  $(i, j)$  have  $i < j$ . (c) Proceeding in node number sequence with  $\nu[start] = 0$  gives  $\nu[1] = 0$ ,  $d[1] = start$ ,  $\nu[2] = 1$ ,  $d[2] = 1$ ,  $\nu[3] = 5$ ,  $d[3] = 2$ ,  $\nu[4] = 5$ ,  $d[4] = 2$ ,  $\nu[5] = 6$ ,  $d[5] = 3$ ,  $\nu[6] = 7$ ,  $d[6] = 5$ ,  $\nu[7] = 11$ ,  $d[7] = 6$ ,  $\nu[8] = 12$ ,  $d[8] = 7$ ,  $\nu[finish] = 13$ ,  $d[finish] = 8$ . (d) Backtracking,  $d[finish] = 8$ ,  $d[8] = 7$ ,  $d[7] = 6$ ,  $d[6] = 5$ ,  $d[5] = 3$ ,  $d[3] = 2$ ,  $d[2] = 1$ ,  $d[1] = start$  to give critical path  $start-1-2-3-5-6-7-8-finish$ . (e) Taking nodes in reverse number sequence with value at  $finish$  of 13 gives late starts: 1 = 0, 2 = 1, 3 = 5, 4 = 7, 5 = 6, 6 = 7, 7 = 11, 8 = 12; Then subtracting late start time minus early start time produces slacks: 1 = 0, 2 = 0, 3 = 0, 4 = 2, 5 = 0, 6 = 0, 7 = 0, 8 = 0

9-24.

(a)

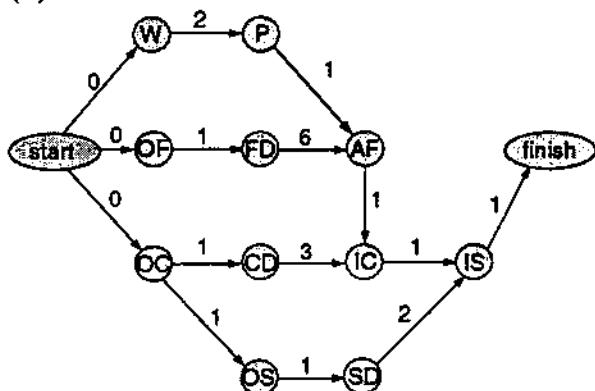


(b) One acyclic node numbering is  $F = 1$ ,  $CS = 2$ ,  $1LB = 3$ ,  $1IW = 4$ ,  $2FL = 5$ ,  $1FN = 6$ ,  $2LB = 7$ ,  $2IW = 8$ ,  $R = 9$ ,  $2FN = 10$ . (c) Proceeding in node number sequence with  $\nu[start] = 0$  gives  $\nu[F] = 0$ ,  $d[F] = start$ ,  $\nu[CS] = 8$ ,  $d[CS] = F$ ,  $\nu[1LB] = 13$ ,  $d[1LB] = CS$ ,  $\nu[1IW] = 13$ ,  $d[1IW] = CS$ ,  $\nu[2FL] = 16$ ,  $d[2FL] = 1LB$ ,  $\nu[1FN] = 19$ ,  $d[1FN] = 2FL$ ,  $\nu[2LB] = 19$ ,  $d[2LB] = 2FL$ ,  $\nu[2IW] = 19$ ,  $d[2IW] = 2FL$ ,  $\nu[R] = 23$ ,  $d[R] = 2LB$ ,  $\nu[2FN] = 25$ ,  $d[2FN] = R$ ,  $finish = 35$ ,  $d[finish] = 2FN$ . (d) Backtrack-

ing,  $d[\text{finish}] = 2FN$ ,  $d[2FN] = R$ ,  $d[R] = 2LB$ ,  $d[2LB] = 2FL$ ,  $d[2FL] = 1LB$ ,  $d[1LB] = CS$ ,  $d[CS] = F$ ,  $d[F] = \text{start}$ , to give critical path  $\text{start} - F - CS - 1LB - 2FL - 2LB - R - 2FN - \text{finish}$  (e) Taking nodes in reverse number sequence with value at  $\text{finish}$  of 35 gives late starts:  $F = 0$ ,  $CS = 8$ ,  $1LB = 13$ ,  $1IW = 19$ ,  $2FL = 16$ ,  $1FN = 23$ ,  $2LB = 19$ ,  $2IW = 20$ ,  $R = 23$ ,  $2FN = 25$ . Then subtracting late start time minus early start time produces slacks:  $F = 0$ ,  $CS = 0$ ,  $1LB = 0$ ,  $1IW = 6$ ,  $2FL = 0$ ,  $1FN = 4$ ,  $2LB = 0$ ,  $2IW = 1$ ,  $R = 0$ ,  $2FN = 0$ .

9-25.

(a)



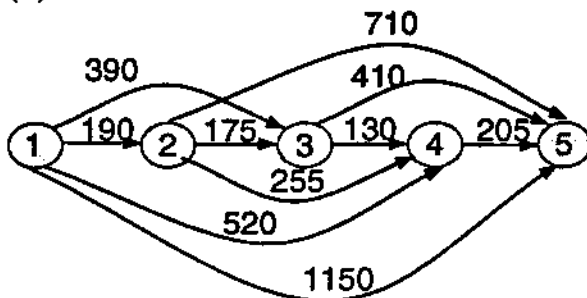
(b) One acyclic node numbering is  $W = 1$ ,  $P = 2$ ,  $OF = 3$ ,  $FD = 4$ ,  $AF = 5$ ,  $OC = 6$ ,  $CD = 7$ ,  $IC = 8$ ,  $OS = 9$ ,  $SD = 10$ ,  $IS = 11$

(c) Proceeding in node number sequence with  $\nu[\text{start}] = 0$  gives  $\nu[W] = 0$ ,  $d[W] = \text{start}$ ,  $\nu[P] = 2$ ,  $d[P] = W$ ,  $\nu[OF] = 0$ ,  $d[OF] = \text{start}$ ,  $\nu[FD] = 1$ ,  $d[FD] = OF$ ,  $\nu[AF] = 7$ ,  $d[AF] = FD$ ,  $\nu[OC] = 0$ ,  $d[OC] = \text{start}$ ,  $\nu[CD] = 1$ ,  $d[CD] = OC$ ,  $\nu[IC] = 8$ ,  $d[IC] = AF$ ,  $\nu[OS] = 1$ ,  $d[OS] = OC$ ,  $\nu[SD] = 2$ ,  $d[SD] = OS$ ,  $\nu[IS] = 9$ ,  $d[IS] = IC$ ,  $\nu[\text{finish}] = 10$ ,  $d[\text{finish}] = IS$ . (d)  $\text{start} - OF - FD - AF - IC - IS - \text{finish}$  (e) Taking nodes in reverse number sequence with value at  $\text{finish}$  of 15 gives late starts:  $W = 9$ ,  $P = 11$ ,  $OF = 5$ ,  $FD = 6$ ,  $AF = 12$ ,  $OC = 9$ ,  $CD = 10$ ,  $IC = 13$ ,  $OS = 11$ ,  $SD = 12$ ,  $IS = 14$ . Then subtracting late start time minus early start time produces slacks:  $W = 9$ ,  $P = 9$ ,  $OF = 5$ ,  $FD = 5$ ,  $AF = 5$ ,  $OC = 9$ ,  $CD = 9$ ,  $IC = 5$ ,  $OS = 10$ ,

$SD = 10$ ,  $IS = 5$ .

9-26. (a) Dynamic programming requires that future decisions depend only on optimal ways to reach prior states. This will be the case in this problem if a period is reached with no backlog and no inventory.

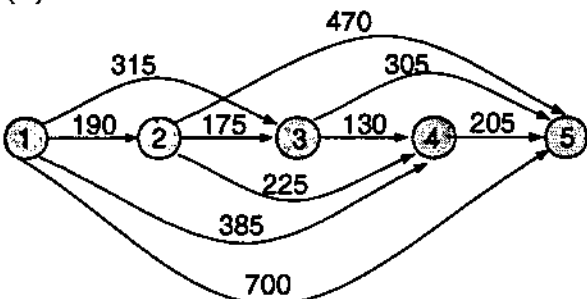
(b)



Arcs correspond to setting up and producing for a series of periods, holding as necessary for periods after the one where production occurs. (c) Every period's demand must be covered. With each arc representing a sequence of periods, a path from 1 to 5 covers all. (d)  $\nu[1] = 0$ ,  $\nu[2] = 190$ ,  $d[2] = 1$ ,  $\nu[3] = 365$ ,  $d[3] = 2$ ,  $\nu[4] = 445$ ,  $d[4] = 2$ ,  $\nu[5] = 650$ ,  $d[5] = 4$ . The implied plan produces in periods 1, 2 and 4 in quantities 30, 35, and 35, respectively. (e) This is the optimal path for node 3, where we will arrive with no inventory. It produces 30 in the first period and 25 in the second.

9-27. (a) Dynamic programming requires that future decisions depend only on optimal ways to reach prior states. This will be the case in this problem if a period is reached with no backlog and no inventory.

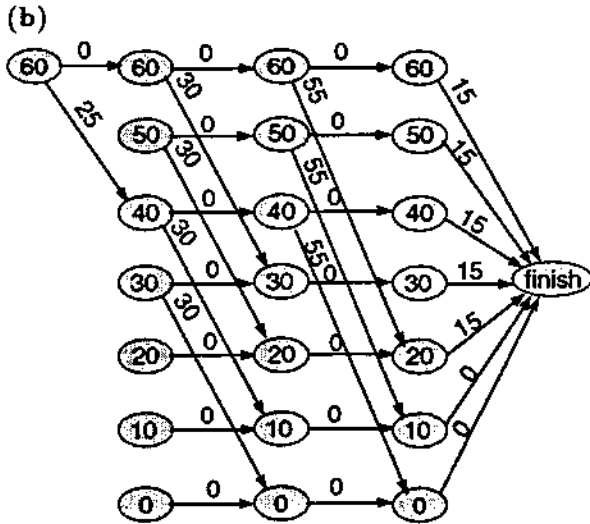
(b)



Arcs correspond to setting up and producing for a series of periods, holding as necessary for

periods after the one where production occurs. (c) Every period's demand must be covered. With each arc representing a sequence of periods, a path from 1 to 5 covers all. (d)  $\nu[1] = 0$ ,  $\nu[2] = 190$ ,  $d[2] = 1$ ,  $\nu[3] = 315$ ,  $d[3] = 1$ ,  $\nu[4] = 385$ ,  $d[4] = 1$ ,  $\nu[5] = 590$ ,  $d[5] = 4$ . The implied plan produces in periods 1 and 4 in quantities 65 and 35, respectively. (e) This is the optimal path to node 3, where we will arrive with no inventory. It produces only in period 1 for the 55 units needed in the first two periods.

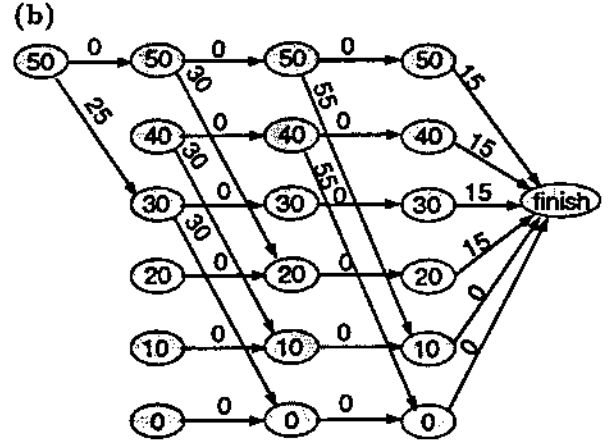
9-28. (a) Decisions must be made about each of the 4 pieces of equipment, and those decisions may depend on how much of the weight limit has already been expended. With all weights multiples of 10 pounds, it is sufficient to have states for just  $w = 10, 20, 30, 40, 50$  and 60.



Horizontal arcs corresponding to not selecting the equipment for that stage, and down-right arcs correspond to selecting the equipment. When the equipment is selected, we gain its probability of use. (c) Starting with stage 1, weight 60, we must find a highest value path through all stages. (d) We take nodes in order, state within stage, setting  $\nu[\ ]$  and  $d[\ ]$  labels. An optimal path takes items 1 and 3 at value 80%.

9-29. (a) Decisions must be made about each of the 4 pieces of equipment, and those decisions may depend on how much of the weight limit has already been expended. With all weights multiples of 10 pounds, it is sufficient to have

states for just  $w = 10, 20, 30, 40, 50$  and 60.



Horizontal arcs corresponding to not selecting the equipment for that stage, and down-right arcs correspond to selecting the equipment. When the equipment is selected, we gain its probability of use. (c) Starting with stage 1, weight 60, we must find a highest value path through all stages. (d) We take nodes in order, state within stage, setting  $\nu[\ ]$  and  $d[\ ]$  labels. An optimal path takes items 1 and 2 at value 55%.

9-30. (a) We will route connections from node 1 to nodes 8, 10, 11 and 12. The most efficient solution would be to route along shortest paths. (b) In order to use dynamic programming based methods we need to solve a one node to all others problem even though we need just 4 paths. With an undirected graph having nonnegative edge weights, the most efficient procedure would be Dijkstra.

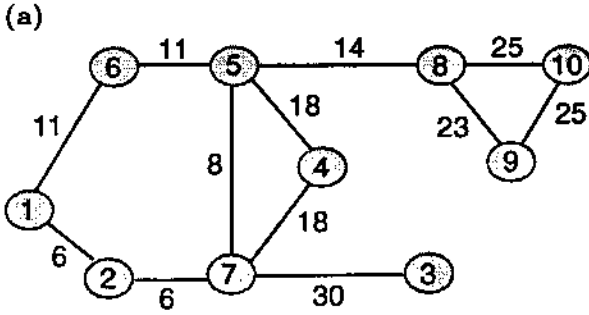
(c) See Table 9.1. Resulting shortest paths to the specified nodes are for 8: 1-2-13-3-8, length 26; for 10: 1-2-13-3-8-10, length 41; for 11: 1-2-13-4-11, length 19; for 12: 1-2-13-3-8-6-12, length 35.

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$	$\nu[6]$	$\nu[7]$	$\nu[8]$	$\nu[9]$	$\nu[10]$	$\nu[11]$	$\nu[12]$	$\nu[13]$
(init)	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
$p = 1$	*	2			27								
$p = 2$		*											6
$p = 13$			11	16									*
$p = 3$			*					26					
$p = 4$				*	23						19		
$p = 11$							22				*		
$p = 7$						31	*						
$p = 5$					*								
$p = 8$						29		*	38	41			
$p = 6$						*						35	
$p = 12$												*	
$p = 9$									*				
$p = 10$										*			
(final)	0	2	11	16	23	29	22	26	38	41	19	35	6

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$	$d[6]$	$d[7]$	$d[8]$	$d[9]$	$d[10]$	$d[11]$	$d[12]$	$d[13]$
$p = 1$		1			1								
$p = 2$													2
$p = 13$			13	13									
$p = 3$								3					
$p = 4$					4						4		
$p = 11$							11						
$p = 7$						7							
$p = 5$													
$p = 8$						8			8	8			
$p = 6$												6	
$p = 12$													
$p = 9$													
$p = 10$													
(final)	-	1	13	13	4	8	11	3	8	8	4	6	2

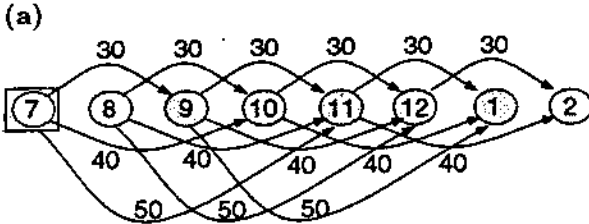
Table 9.1: Dijkstra Computations for Exercise 9-30

9-31.



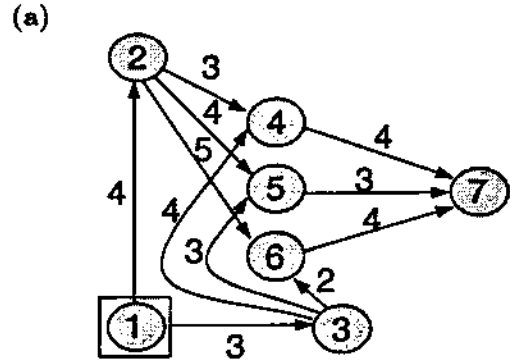
Routings should be along a least time path from node 1 to all others, with edge lengths equal to the maximum of times on nodes at its ends. (b) We need paths from one node to all others, edge weights are nonnegative, and the graph is undirected. Dijkstra is the most efficient choice available. (c) See Table 9.2. Optimal paths are 1-2, length 6; 1-2-7-3, length 42; 1-2-7-4, length 30; 1-2-7-5, length 20; 1-6, length 11; 1-2-7, length 12; 1-2-7-5-8, length 34; 1-2-7-5-8-9, 57; 1-2-7-5-8-10, length 59

9-32.



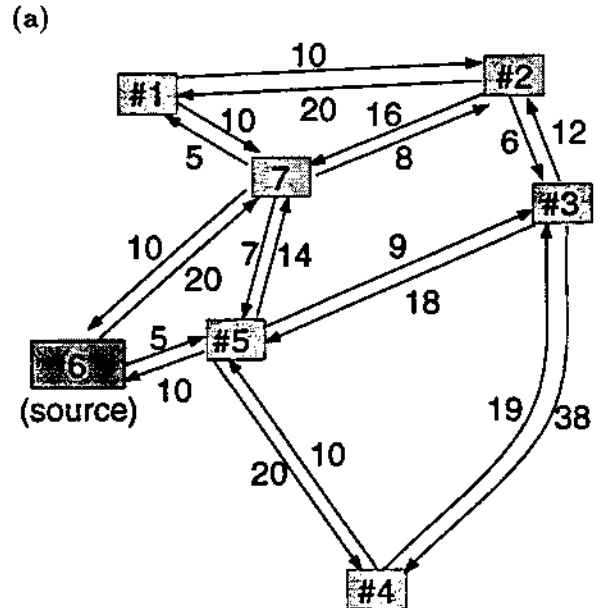
The least total cost path from 7:00 P.M. to 2:00 A.M. is the least expensive way to provide staffing across all hours. (b) The digraph is acyclic because every arc goes from some hour to a later one. (c) The best choice is shortest acyclic algorithm 9D when the graph is acyclic. We only need a path from 7:00 to 2:00, but dynamic programming methods require us to find best paths from 7:00 to all other times. (d) Taking nodes in time sequence,  $\nu[7] = 0, d[7] = -$ ,  $\nu[8] = \infty, d[8] = 0, \nu[9] = 30, d[9] = 7$ ,  $\nu[10] = 40, d[10] = 7, \nu[11] = 50, d[11] = 7$ ,  $\nu[12] = 70, d[12] = 10, \nu[1] = 80, d[1] = 11$ ,  $\nu[2] = 90, d[2] = 11$ . Thus a best schedule is 7-11-2 with one 4-hour and one 3-hour at cost \$90.

9-33.



We need a shortest path from first station 1 to last station 7, with arc length measured by the rectilinear distance between points. We only need one path, but dynamic programming methods require us to find best paths from 1 to all other nodes. (b) The digraph is acyclic because each possible movement is from a node to a higher numbered one. (c) The best choice is shortest acyclic algorithm 9D when the graph is acyclic. (d) Taking nodes in number sequence,  $\nu[1] = 0, d[1] = 0, \nu[2] = 4, d[2] = 1, \nu[3] = 3, d[3] = 1, \nu[4] = 7, d[4] = 3, \nu[5] = 6, d[5] = 3, \nu[6] = 5, d[6] = 3, \nu[7] = 9, d[7] = 6$ . Thus a best routing is 1-3-6-7 (or 1-3-5-7) with length 9 times the gridsize.

9-34.



We need shortest roll out routes from the shed

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$	$\nu[6]$	$\nu[7]$	$\nu[8]$	$\nu[9]$	$\nu[10]$
(init)	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
$p = 1$	*	6				11				
$p = 2$		*					12			
$p = 6$					22	*				
$p = 7$			42	30	20		*			
$p = 5$					*			34		
$p = 4$				*						
$p = 8$								*	57	59
$p = 3$			*							
$p = 9$									*	
$p = 10$										*
(final)	0	6	42	30	20	11	12	34	57	59

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$	$d[6]$	$d[7]$	$d[8]$	$d[9]$	$d[10]$
$p = 1$		1				1				
$p = 2$							2			
$p = 6$					6					
$p = 7$			7	7	7					
$p = 5$							5			
$p = 4$										
$p = 8$								8		8
$p = 3$										
$p = 9$										
$p = 10$										
(final)	-	1	7	7	7	1	2	5	8	8

Table 9.2: Dijkstra Computations for Exercise 9-31

to locations #1 through #5. There are two opposed arcs between any pair of points, an uphill one with the indicated energy and a downhill one with half the energy. (b) The digraph is not acyclic because each pair of opposed arcs forms a dicycle. (c) For a shortest one to all others problem with nonnegative costs and dicycles, the best choice is Dijkstra Algorithm 9C.

(d) Number the shed 6 and the unlabeled intersection 7.

	$\nu[1]$	$\nu[2]$	$\nu[3]$	$\nu[4]$	$\nu[5]$	$\nu[6]$	$\nu[7]$
(init)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$
$p = 6$					5	*	20
$p = 5$			14	25	*		19
$p = 3$		26	*				
$p = 7$	24						*
$p = 1$	*						
$p = 4$				*			
$p = 2$		*					
(final)	24	26	14	25	5	0	19

	$d[1]$	$d[2]$	$d[3]$	$d[4]$	$d[5]$	$d[6]$	$d[7]$
$p = 6$					6		6
$p = 5$			5	5			5
$p = 3$		3					
$p = 7$	7						
$p = 1$							
$p = 4$							
$p = 2$							
(final)	7	3	5	5	6	-	5

Optimal paths are 6 - 5 - 7 - 1, length 24; 6 - 5 - 3 - 2, length 26; 6 - 5 - 3, length 19; 6 - 5 - 4, length 25; 6 - 5, length 5