Chapter 4

Exercise Solutions

4-1. (a) $x_j \triangleq$ cases shipped to region j, max $1.60x_1 + 1.40x_2 + 1.90x_3 + 1.20x_4$ (total profit), s.t. $\sum_{j=1}^{4} x_j = 1200$ (60% of 2000 available), $310 \leq x_1 \leq 434$ (50% to 70% of NE demand), $245 \leq x_2 \leq 343$ (50% to 70% of SE demand), $255 \leq x_3 \leq 357$ (50% to 70% of MW demand), $190 \leq x_4 \leq 266$ (50% to 70% of W demand)
(b) $x_1^* = \text{NE} = 408$, $x_2^* = \text{SE} = 245$, $x_3^* = \text{MW} = 357$, $x_4^* = \text{W} = 190$

4-2. (a) $x_{i,j} \triangleq$ hours of designer i on project j, and let $s_{i,j} \triangleq$ scores in table, $r_j \triangleq$ requirements in table;
max $\sum_{i=1}^{3}\sum_{j=1}^{4} s_{i,j}x_{i,j}$ (max total score), s.t. $\sum_{j=1}^{4} x_{i,j} = 80$, $i = 1,\ldots,3$ (80 hours for each designer i), $\sum_{i=1}^{3} x_{i,j} = r_j$, $j = 1,\ldots,4$ (requirements for each project j), $x_{i,j} \geq 0$, $i = 1,\ldots,3, j = 1,\ldots,4$
(b) Nonzero values are $x_{1,1}^* = 70$, $x_{1,2}^* = 10$, $x_{2,2}^* = 40$, $x_{2,3}^* = 5$, $x_{2,4}^* = 35$, $x_{3,4}^* = 80$

4-3. (a) $x_j \triangleq$ fraction of ingredient j;
min $200x_1 + 150x_2 + 100x_3 + 75x_4$ (min total cost), s.t. $\sum_{j=1}^{4} x_j = 1$ (blend fractions sum to 1), $60x_1 + 80x_2 + 55x_3 + 40x_4 \geq 60$ (min % protein), $50x_1 + 70x_2 + 40x_3 + 100x_4 \leq 60$ (max % fat), $90x_1 + 30x_2 + 60x_3 + 80x_4 \geq 60$ (min % fiber)
(b) The 3 main inequalities, which control properties of the blend.
(c) $x_1^* = \text{oats} = 157$, $x_2^* = \text{corn} = 271$, $x_3^* = \text{alfalfa} = 401$, $x_4^* = \text{hulls} = 171$

4-4. (a) $x_j \triangleq$ the fraction of gasoline j;
min $48x_1 + 43x_2 + 58x_3 + 46x_4$ (min total cost), s.t. $\sum_{j=1}^{4} x_j = 1$ (blend fractions sum to 1), $85 \leq 99x_1 + 70x_2 + 78x_3 + 91x_4 \leq 90$ (1st quality index of blend within limits), $270 \leq 210x_1 + 335x_2 + 280x_3 + 265x_4 \leq 280$ (2nd quality index of blend within limits), $x_j \geq 0$, $j = 1,\ldots,4$
(b) The 4 main inequalities, which control properties of the blend.
(c) $x_1^* = .176$, $x_2^* = .353$, $x_3^* = .000$, $x_4^* = .471$

4-5. (a) $45 \sum_{i=1}^{m} x_{i,j} \leq \sum_{i=1}^{m} a_{i,11}x_{i,j} \leq 48 \sum_{i=1}^{m} x_{i,j}$, $j = 1,\ldots,n$
(b) $90\sum_{i=1}^{m} x_{i,14} \leq \sum_{i=1}^{m} a_{i,k}x_{i,14} \leq 95 \sum_{i=1}^{m} x_{i,14}$, $k = 5,\ldots,9$
(c) $\sum_{i=1}^{m} a_{i,15}x_{i,15} \geq 116\sum_{i=1}^{m} x_{i,15}$
(d) $\sum_{i=1}^{m} a_{i,8}x_{i,j} \leq 87\sum_{i=1}^{m} x_{i,j}$, $j = 1,\ldots,n$
(e) $7x_{1,j} \leq 3\sum_{i=2}^{m} x_{i,j}$, $j = 6,\ldots,11$
(f) $3x_{4,j} = 2x_{7,j}$, $j = 1,\ldots,n$
(g) $3\sum_{i=3}^{6}\sum_{j=1}^{n} x_{i,j} \geq \sum_{i=1}^{m}\sum_{j=1}^{n} x_{i,j}$
(h) $\sum_{j=1}^{n} x_{13,j} \leq .05\sum_{i=1}^{m}\sum_{j=1}^{n} x_{i,j}$

4-6. LP's are much more tractable than ILP's, and the error resulting from rounding off fractions is usually minimal when decision variable magnitudes are greater than 1 or 2.

4-7. (a) $x_j \triangleq$ the number of cuts with pattern j; min $.34x_1 + .22x_2 + .27x_3$ (min total waste), s.t. $2x_1 + 1x_3 \geq 37$ (large disk production), $5x_2 + 3x_3 \geq 211$ (small disk production), $x_j \geq 0$, $j = 1,\ldots,4$
(b) $x_1^* = 147.4$, $x_2^* = 0.0$, $x_3^* = 21.1$

4-8. (a) $x_j \triangleq$ units of model j;
max $16x_1 + 9x_2 + 27x_3$ (max total profit), s.t. $.10x_1 + .10x_2 + .25x_3 \leq 20$ (molder capacity), $.35x_1 + .15x_2 + .40x_3 \leq 60$ (decorator capacity), $.08x_1 + .03x_2 + .05x_3 \leq 20$ (packager capacity), $x_1, x_2, x_3 \geq 0$
(b) $x_1^* = \text{santas} = 147.4$, $x_2^* = \text{trees} = 0.0$, $x_3^* = \text{houses} = 21.1$

4-9. (a) max $30x_1 + 45x_2$ (max total profit), s.t. $.30x_1 + .30x_2 + .10x_3 + .15x_4 + .50x_5 \leq 80$ (labor hours available), $1.5x_3 + 2.5x_4 \leq 500$ (leg stock available), $x_3 = 4x_1$ (short leg balance), $x_4 = 4x_2$ (long leg balance), $x_5 = x_1 + x_2$ (table tops balance), $x_j \geq 0,\ j = 1, \ldots, 5$
(b) The 3 equalities, which equate needed and available numbers of parts.
(c) $x_1^* = 27.8$, $x_2^* = 33.3$, $x_3^* = 111.1$, $x_4^* = 133.3$, $x_5^* = 61.1$

4-10. (a) max $5x_1 + 7x_2$ (max total profit), s.t. $.25x_1 + .30x_2 \leq 200$ (assembly capacity), $.005x_3 + .007x_4 + .002x_5 \leq 40$ (fabrication capacity), $x_3 = 3x_1$ (standard separator balance), $x_4 = 3x_2$ (extra-long separator balance), $x_5 = 10x_1 + 18x_2$ (cross piece balance), $x_j \geq 0,\ j = 1, \ldots, 5$
(b) The 3 equalities, which equate needed and available numbers of parts.
(c) $x_1^* = 0.0$, $x_2^* = 666.7$, $x_3^* = 0.0$, $x_4^* = 2000$, $x_5^* = 12000$

4-11. (a) $\sum_{p=1}^{n} x_{i,p} = \sum_{p=1}^{n} d_{i,p} + \sum_{k=1}^{m} \sum_{p=1}^{n} a_{i,k} x_{k,p},\ i = 1, \ldots, m$
(b) $\sum_{q=1}^{n} x_{i,q,p} = d_{i,p} + \sum_{k=1}^{m} \sum_{q=1}^{n} a_{i,k} x_{k,p,q},\ i = 1, \ldots, m,\ p = 1, \ldots, n$

4-12. (a) $x_1 \triangleq$ number with 5 days starting Sunday,..., $x_7 \triangleq$ number with 5 days starting Saturday;
min $\sum_{j=1}^{7} x_j$ (min total officers), s.t. $x_1 + x_4 + x_5 + x_6 + x_7 \geq 8$ (cover Sun), $x_1 + x_2 + x_5 + x_6 + x_7 \geq 6$ (cover Mon), $x_1 + x_2 + x_3 + x_6 + x_7 \geq 6$ (cover Tue), $x_1 + x_2 + x_3 + x_4 + x_7 \geq 6$ (cover Wed), $x_1 + x_2 + x_3 + x_4 + x_5 \geq 6$ (cover Thu), $x_2 + x_3 + x_4 + x_5 + x_6 \geq 10$ (cover Fri), $x_3 + x_4 + x_5 + x_6 + x_7 \geq 10$ (cover Sat), $x_j \geq 0,\ j = 1, \ldots, 7$
(b) All main constraints, which assure coverage on different days.
(c) $x_1^* = 0.0$, $x_2^* = .67$, $x_3^* = 2.0$, $x_4^* = 2.67$, $x_5^* = 2.0$, $x_6^* = 2.67$, $x_7^* = .67$

4-13. (a) $x_1 \triangleq$ number starting 5 A.M.,..., $x_6 \triangleq$ number starting 10 A.M.,
min $28x_1 + 28x_2 + 28x_3 + 24x_4 + 24x_5 + 24x_6$ (min total payroll), $x_1 \geq 2$ (cover 6 A.M.), $x_1 + x_2 \geq 3$ (cover 7 A.M.), $x_1 + x_2 + x_3 \geq 5$ (cover 8 A.M.), $x_1 + x_2 + x_3 + x_4 \geq 5$ (cover 9 A.M.), $x_2 + x_3 + x_4 + x_5 \geq 3$ (cover 10 A.M.), $x_3 + x_4 + x_5 + x_6 \geq 2$ (cover 11 A.M.), $x_4 + x_5 + x_6 \geq 4$ (cover noon), $x_5 + x_6 \geq 6$ (cover 1 P.M.), $x_6 \geq 3$ (cover 2 P.M.), $x_j \geq 0,\ j = 1, \ldots, 6$
(b) All main constraints, which assure coverage at different times.
(c) $x_1^* = 5$, $x_2^* = 0$, $x_3^* = 0$, $x_4^* = 0$, $x_5^* = 3$, $x_6^* = 3$

4-14. (a) $x_{j,t} \triangleq$ investment in option j, year t;
max $1.05x_{1,4} + 1.12x_{2,3} + 1.21x_{3,1}$ (max final value), s.t. $10 = x_{1,1} + x_{2,1} + x_{3,1}$ (year 1 balance), $1.05x_{1,1} + 10 = x_{1,2} + x_{2,2}$ (year 2 balance), $1.05x_{1,2} + 1.12x_{2,1} + 10 = x_{1,3} + x_{2,3}$ (year 3 balance), $1.05x_{1,3} + 1.12x_{2,2} = x_{1,4}$ (year 4 balance), all variables nonnegative
(b) All main constraints, which equate available and invested funds.
(c) 4 years, the fixed time period over which decisions are to be made.
(d) Nonzero values are $x_{2,1}^* = x_{2,2}^* = 10$, $x_{2,3}^* = x_{1,4}^* = 21.2$

4-15. (a) $x_t \triangleq$ production in quarter t, $z_t \triangleq$ ending inventory in quarter t;
min $15 \sum_{t=1}^{4} z_t$ (min total inventory cost), s.t. $z_4 + x_1 = 2800 + z_1$ (quarter 1 inventory balance), $z_1 + x_2 = 500 + z_2$ (quarter 2 inventory balance), $z_2 + x_3 = 100 + z_3$ (quarter 3 inventory balance), $z_3 + x_4 = 850 + z_4$ (quarter 4 inventory balance), $x_t \leq 1200\ t = 1, \ldots, 4$ (capacity in quarter t), all variables nonnegative
(b) All main constraints, which equate available inventory and production to demand and future inventory.
(c) Inventories wrap around at the end of the 4 quarter period so operations continue forever.
(d) $x_1^* = x_3^* = x_4^* = 1200$, $x_2^* = 650$, $z_1^* = 0$, $z_2^* = 150$, $z_3^* = 1250$, $z_4^* = 1600$

4-16. (a) $\sum_{i=1}^{m} a_{i,k} x_{i,t} \leq b_k,\ k = 1, \ldots, q,\ t = 1, \ldots, n$
(b) $\sum_{i=1}^{m} v_i z_{i,t} \leq 200,\ t = 1, \ldots, n$
(c) $x_{i,1} = d_{i,1} + z_{i,1},\ i = 1, \ldots, m$
(d) $z_{i,t-1} + x_{i,t} = d_{i,t} + z_{i,t},\ i = 1, \ldots, m,\ t = 2, \ldots, n-1$

4-17. (a) $w_j^+ \triangleq$ over-estimation on point j, $w_j^- \triangleq$ under-estimation on point j;
min $\sum_{j=1}^{4} \left(w_j^+ + w_j^-\right)$ (min total deviation), s.t. $\beta_0 + 2\beta_1 = 1 + w_1^+ - w_1^-$ (deviation for level 2),

$\beta_0 + 3\beta_1 = 3 + w_2^+ - w_2^-$ (deviation for level 3), $\beta_0 + 5\beta_1 = 3 + w_3^+ - w_3^-$ (deviation for level 5), $\beta_0 + 7\beta_1 = 5 + w_4^+ - w_4^-$ (deviation for level 7), all variables nonnegative

(b) $\beta_0^* = .000$, $\beta_1^* = .714$, nonzero over $w_1^{+*} = .429$, $w_3^{+*} = .571$, nonzero under $w_2^{-*} = .857$

(c) $w_1^{+*} + w_1^{-*} = .429$, $w_2^{+*} + w_2^{-*} = .857$, $w_3^{+*} + w_3^{-*} = .571$, $w_4^{+*} + w_4^{-*} = .000$

4-18. **(a)** $x \triangleq$ box location, $w_j^+ \triangleq$ right or x-length to j, $w_j^- \triangleq$ left or x-length to j;
min $\sum_{j=1}^{4} \left(w_j^+ + w_j^-\right)$ (max total x cable, y being fixed), s.t. $5 = x + w_1^+ - w_1^-$ (x-displacement to (5,8)), $10 = x + w_2^+ - w_2^-$ (x-displacement to (10,15)), $25 = x + w_3^+ - w_3^-$ (x-displacement to (25,10)), all variables nonnegative

(b) $x^* = 10$, nonzero runs $w_3^{+*} = 15$, $w_1^{-*} = 5$

(c) (optimal x-displacements) + (fixed y-displacements) = $(5 + 15) + (8 + 15 + 10) = 53$ feet

4-19. **(a)** $z \triangleq$ largest deviation, $w_j^+ \triangleq$ over-estimation on point j, $w_j^- \triangleq$ under-estimation on point j;
min z (minmax deviation), s.t. $\beta_0 + 2\beta_1 = 1 + w_1^+ - w_1^-$ (deviation at level 2), $\beta_0 + 3\beta_1 = 3 + w_2^+ - w_2^-$ (deviation at level 3), $\beta_0 + 5\beta_1 = 3 + w_3^+ - w_3^-$ (deviation at level 5), $\beta_0 + 7\beta_1 = 5 + w_4^+ - w_4^-$ (deviation at level 7), $z \geq w_j^+$, $j = 1, \ldots, 4$ (max deviation $\leq j$ over), $z \geq w_j^-$, $j = 1, \ldots, 4$ (max deviation $\leq j$ under), all variables nonnegative

(b) $\beta_0^* = .333$, $\beta_1^* = .667$, $z^* = .667$, nonzero over $w_1^{+*} = w_3^{+*} = .667$, nonzero under $w_2^{-*} = .667$

(c) $w_1^{+*} + w_1^{-*} = .667$, $w_2^{+*} + w_2^{-*} = .667$, $w_3^{+*} + w_3^{-*} = .667$, $w_4^{+*} + w_4^{-*} = .000$

4-20. **(a)** $x \triangleq$ box location, $z \triangleq$ longest cable, $w_j^+ \triangleq$ right length to j, $w_j^- \triangleq$ left length to j;
min z (min max cable), s.t. $5 = x + w_1^+ - w_1^-$ (x-displacement to (5,8)), $10 = x + w_2^+ - w_2^-$ (x-displacement to (10,15)), $25 = x + w_3^+ - w_3^-$ (x-displacement to (25,10)), $z \geq 8 + w_1^+ + w_1^-$ (max $\geq$ cable to (5,8)), $z \geq 15 + w_2^+ + w_2^-$ (max $\geq$ cable to (10,15)), $z \geq 10 + w_3^+ + w_3^-$ (max $\geq$ cable to (25,10)), all variables nonnegative

(b) $x^* = 15$, $z^* = 20$, nonzero runs $w_3^{+*} = 10$, $w_1^{-*} = 10$, $w_2^{-*} = 5$

(c) (optimal x-displacements) + (fixed y-displacements) + $(10 + 10 + 5) + (8 + 15 + 10) =$ 58 feet

4-21. **(a)** min $\sum_{j=1}^{5} c_j x_j$ (min total cost), s.t. $\sum_{j=1}^{5} a_{i,j} x_j \geq r_i$, $i = 1, \ldots, 7$ (meet requirements for size i); $x_j \leq u_j$, $j = 1, \ldots, 5$ (available from supplier j); all variables nonnegative; where the $a_{i,j}$ are the yield fractions in the table, c_j the costs, and u_j the availabilities; the r_i are the given requirements.

(b) $x_1^* = 18.75$, $x_2^* = 125.00$, $x_3^* = 150.00$, $x_4^* = 650.00$, $x_5^* = 0.00$

4-22. **(a)** min $\sum_{j=1}^{5} \sum_{q=1}^{4} .05 v_j h_{j,q}$ (min total inventory holding cost), s.t. $\sum_{j=1}^{5} t_j x_{j,q} \leq 1150$, $q = 1, \ldots, 4$ (production time capacity in quarter t); $h_{1,q-1} + x_{1,q} = x_{4,q} + h_{1,q}$, $q = 1, \ldots, 4$ (element 1 balance in quarter q); $h_{2,q-1} + x_{2,q} = 4x_{4,q} + 8x_{5,q} + h_{2,q}$, $q = 1, \ldots, 4$ (element 2 balance in quarter q); $h_{3,q-1} + x_{3,q} = x_{5,q} + h_{3,q}$, $q = 1, \ldots, 4$ (element 3 balance in quarter q); $h_{4,q-1} + x_{4,q} = x_{5,q} + h_{4,q}$, $q = 1, \ldots, 4$ (element 4 balance in quarter q); $h_{5,q-1} + x_{5,q} = r_q + h_{5,q}$, $q = 1, \ldots, 4$ (element 5 balance in quarter q); all variables nonnegative; where v_j and t_j are the values in the table, the r_q are the quarterly demands, and $q - 1$ is taken as 4 when $q = 1$.

(b) Nonzeros $x_{1,1}^* = x_{1,2}^* = x_{1,3}^* = 833.3$, $x_{1,4}^* = 500.0$, $x_{2,1}^* = x_{2,2}^* = x_{2,3}^* = 10000.0$, $x_{2,4}^* = 6000.0$, $x_{3,1}^* = x_{3,2}^* = x_{3,3}^* = 833.3$, $x_{3,4}^* = 500.0$, $x_{4,1}^* = x_{4,2}^* = x_{4,3}^* = 833.3$, $x_{4,4}^* = 500.0$, $x_{5,1}^* = x_{5,2}^* = x_{5,3}^* = 833.3$, $x_{5,4}^* = 500.0$, $h_{5,1}^* = 533.3$, $h_{5,2}^* = 166.7$, $h_{5,4}^* = 200.0$

4-23. **(a)** min $\sum_{j=1}^{7} \left(\underline{c}_j + (t_j - \underline{t}_j)(\overline{c}_j - \underline{c}_j) / (\overline{t}_j - \underline{t}_j)\right)$ (min sum of interpolated task costs), s.t. $s_3 \geq s_2 + t_2$ (2 precedes 3); $s_4 \geq s_1 + t_1$ (1 precedes 4); $s_4 \geq s_2 + t_2$ (2 precedes 4); $s_5 \geq s_3 + t_3$ (3 precedes 5); $s_6 \geq s_3 + t_3$ (3 precedes 6); $s_7 \geq s_4 + t_4$ (4 precedes 7); $s_j + t_j \leq 40$, $j = 1, \ldots, 7$ (task j complete within 40 days); $s_j \geq 0$, $j = 1, \ldots, 7$; $t_j \geq 0$, $j = 1, \ldots, 7$; where $\underline{t}_j$ and $\overline{t}_j$ are the given min and max times for task j, and $\underline{c}_j$ and $\overline{c}_j$ are the corresponding min and max costs.

(b) $s_1^* = s_2^* = 0$, $s_3^* = 8$, $s_4^* = 12$, $s_5^* = s_6^* = 24$, $s_7^* = 32$, $t_1^* = 12$, $t_2^* = t_7^* = 8$, $t_3^* = t_5^* = t_6^* = 16$, $t_4^* = 20$

4-24. min $\sum_{i=1}^{4}\sum_{j=i}^{4} c_{i,j}x_{i,j}$ (min total storage cost), s.t. $\sum_{j=i}^{4} x_{i,j} = b_i$, $i = 1,\ldots,4$ (all title i's are stored); $3x_{1,1} + 2(x_{1,2} + x_{2,2}) + (x_{1,3} + x_{2,3} + x_{3,3}) \leq 500$ (up-to-100 bins available); $\sum_{i=1}^{4} x_{i,4} \leq 2000$ (large bins available); all variables nonnegative

4-25. min $\sum_{c=1}^{12} m_c x_c + b\sum_{d=1}^{12} y_d$ (min total maintenance cost and bulk sale loss); s.t. $\sum_{c=1}^{12} p_{d,c}x_c = r_d + y_d$, $d = 1,\ldots,12$ (milk balance in month d); all variables nonnegative

4-26. min $\sum_{m=1}^{350}\sum_{p=60}^{70} pw_m x_{m,p}$ (min total waste), s.t. $\sum_{m=1}^{350}\sum_{p=60}^{70} pa_{i,m}x_{m,p} \geq d_i$, $i = 1,\ldots,75$ (satisfy demand for fabric part i); all variables nonnegative

4-27. min $\sum_{i=1}^{24}\sum_{m=1}^{8}\sum_{j=1}^{113} c_{i,m,j}x_{i,m,j}$ (min total cost), s.t. $\sum_{m=1}^{8}\sum_{j=1}^{113} x_{i,m,j} \leq s_i$, $i = 1,\ldots,24$ (supply limit in mining region i); $\sum_{i=1}^{24} a_{i,m}x_{i,m,j} = d_{m,j}$, $m = 1,\ldots,8$, $j = 1,\ldots,113$ (demand for coal type m in region j); all variables nonnegative

4-28. min $\sum_{t=0}^{23} c_t x_t$ (min total daily payroll cost), s.t. $\sum_{j=t-8}^{t} x_t - \sum_{j=t-5}^{t-3} y_{j,t} \geq r_t$, $t = 0,\ldots,23$ (cover hour t); $\sum_{i=t+4}^{t+6} y_{t,i} = x_t$, $t = 0,\ldots,23$ (shift t lunches); all variables nonnegative; where hour subscripts $j,t<0$ are interpreted as $j+24, t+24$ and those $j,t>23$ are interpreted as $j-24, t-24$

4-29. min $\sum_{t=1}^{18}(d_t^+ + d_t^-)$ (min total over and under satisfaction of demand), s.t. $s_{t-1} + x_t = w_t + s_t$, $t = 1,\ldots,18$ (define $s_0 = 120$) (storage balance in time period t); $w_t - d_t^+ + d_t^- = r_t$, $t = 1,\ldots,18$ (actual vs. ideal deviations in time period t); $s_t \leq u$, $t = 1,\ldots,18$ (storage limit in time period t); all variables nonnegative

4-30. max $\sum_{i=1}^{20}\sum_{j=1}^{5} p_{i,j}x_{i,j}$ (max total present worth of revenues), s.t. $\sum_{i=1}^{20} a_i x_{i,j} \leq 150$, $j = 1,\ldots,5$ (sector j at most 150 thousand sqft); $\underline{n}_i \leq \sum_{j=1}^{5} x_{i,j} \leq \overline{n}_i$, $i = 1,\ldots,20$ (number of stores of type i between specified min and max); $\underline{f}_i \leq \sum_{j=1}^{5} a_i x_{i,j} \leq \overline{f}_i$, $i = 1,\ldots,20$ (total area for stores of type i between specified min and max); $\sum_{i=1}^{20}\sum_{j=1}^{5} c_i a_i x_{i,j} \leq b$ (total finishing allowances within budget); all variables nonnegative.

4-31. min $\sum_{i=1}^{m}\sum_{s'=1}^{p}\sum_{s<s'} c_{i,s',s}y_{i,s',s}$ (min total misapplication trim loss cost), s.t. $\sum_{i=1}^{m}\sum_{s=1}^{p} x_{i,j,s} \leq 1$, $j = 1,\ldots,n$ (furnace j at most 100% allocated); $\sum_{j=1}^{n} a_{j,s}x_{i,j,s} - \sum_{s'<s} y_{i,s,s'} + \sum_{s'>s} y_{i,s',s} = d_{i,s}$, $i = 1,\ldots,m$, $s = 1,\ldots,p$ (meet net demand for alloy i and size s); all variables nonnegative

4-32. min $\sum_{\ell=2}^{5}\ell\sum_{h=0}^{23} x_{h,\ell}$ (min total hours worked), s.t. $\sum_{\ell=2}^{5}\sum_{k=h-(\ell-1)}^{h} x_{k,\ell} \geq r_h$, $h = 0,\ldots,23$ (provide coverage in hour h); $\sum_{h=0}^{23} x_{h,\ell} \leq b_\ell$, $\ell = 2,\ldots,5$ (total working shift length ℓ within number willing), all variables nonnegative; negative hour subscripts k are interpreted as $k+24$

4-33. min $\sum_{i=1}^{m}\sum_{j=1}^{n}\left(d_{i,j}^+ + d_{i,j}^-\right)$, (min total gray-scale deviations), s.t. $b_{i,j}x_{i,j} + d_{i,j}^+ - d_{i,j}^- = g_{i,j}$, $i = 1,\ldots,m$, $j = 1,\ldots,n$ (predicted vs. transmitted for pixel (i,j)); all variables nonnegative

4-34. max $\sum_{j=1}^{38} x_j$ (max total allowed to enter), s.t. $\sum_{j=1}^{38} f_{i,j}x_j \leq b_i$, $i = 1,\ldots,23$ (at most capacity on segment i); $q_j + d_j - x_j \leq u_j$ (ending queue at point j within limit), $j = 1,\ldots,38$; $x_j \leq q_j + d_j$ (entering at j at most queue plus demand), $j = 1,\ldots,38$, all variables nonnegative

4-35. min $\sum_{i=1}^{18}\sum_{j=1}^{18} f_{i,j}\left(d_{i,j}^+ + d_{i,j}^-\right)$ (min total flow-distance), s.t. $\underline{x}_i \leq x_{i+1} - x_i$, $i = 1,\ldots,18$ (cell i width at least specified min); $x_{19} \leq 1000$ (last cell within building width); $\underline{y}_i \leq y_i \leq 194$, $i = 1,\ldots,18$ (cell i depth at least lower limit and at most building size); $\underline{p}_i \leq 2(x_{i+1} - x_i + y_i)$, $i = 1,\ldots,18$ (cell i perimeter at least lower limit); $.5(x_i + x_{i+1}) - .5(x_j + x_{j+1}) = d_{i,j}^+ - d_{i,j}^-$, $i,j = 1,\ldots,18$ (I/O station to I/O station distances between cells i and j); all variables nonnegative

4-36. max $\sum_{i=1}^{8}\sum_{k=1}^{25} r_{i,k}x_{i,k} + \sum_{i=1}^{8} a_i h_i$ (max total profit + ending asset value), s.t. $\underline{h}_i + q_i - \sum_{k=1}^{25} x_{i,k} = h_i$, $i = 1,\ldots,8$ (inventory balance at pile i); $\underline{s}_k \leq \sum_{i=1}^{8} x_{i,k} \leq \overline{s}_k$, $k = 1,\ldots,25$ (customer k shipped an amount between specified min and max contracted); $\underline{p}_k\sum_{i=1}^{8} x_{i,k} \leq \sum_{i=1}^{8} b_i x_{i,k} \leq \overline{p}_k\sum_{i=1}^{8} x_{i,k}$, $k = 1,\ldots,25$ (customer k BPL blend within specified min and max contracted); all variables nonnegative

4-37. min α (min step to a collision), s.t. $a_i x + b_i y + c_i z \leq d_i$, $i = 1, \ldots, 19$ (collision point (x, y, z) should be within the object, i.e. satisfy all defining constraints i); $p_j(x - \alpha\,\Delta x) + q_j(y - \alpha\,\Delta y) + r_j(z - \alpha\,\Delta z) \leq s_j$, $j = 1, \ldots, 12$ (collision point (x, y, z) should be within the moving robot link, i.e. satisfy all defining constraints j at its original position); $\alpha \geq 0$. Infeasible indicates no common point within a nonnegative step α and thus no collision.

4-38. max $\sum_{f=1}^{10} \sum_{m=1}^{20} p_{f,m} x_{f,m}$ - $\sum_{i=1}^{3} c_i y_i$ (max total profit less labor cost), s.t. $\sum_{m=1}^{20} x_{f,m} \leq b_f$, $f = 1, \ldots, 10$ (availability of species f); $\sum_{f=1}^{10} a_{f,m} x_{f,m} \leq u_m$, $m = 1, \ldots, 20$ (maximum total sales at market m); $\sum_{f=1}^{10} \sum_{m=1}^{20} h_{f,m,i} x_{f,m} = y_i$, $i = 1, \ldots, 3$ (work station i hours balance); $y_i \leq q_i$, $i = 1, \ldots, 3$ (work station i hours available); all variables nonnegative

4-39. max $\sum_{i=1}^{10} \sum_{j=1}^{25} \sum_{k=1}^{15} \sum_{\ell=1}^{8} p_{i,j,k,\ell} r_k x_{i,j,k,\ell}$ (max expected total target value killed) s.t. $\sum_{j=1}^{25} \sum_{k=1}^{15} \sum_{\ell=1}^{8} x_{i,j,k,\ell} \;/\; s_{i,j,k,\ell} \leq a_i + y_i$, $i = 1, \ldots, 10$ (aircraft type i required within current + new); $\sum_{i=1}^{10} \sum_{k=1}^{15} \sum_{\ell=1}^{8} b_{i,j,k,\ell} x_{i,j,k,\ell} \leq m_j + z_j$, $j = 1, \ldots, 25$ (munitions type j required within current + new); $\sum_{i=1}^{10} \sum_{j=1}^{25} p_{i,j,k,\ell} x_{i,j,k,\ell} \leq t_{k,\ell}$, $k = 1, \ldots, 15$, $\ell = 1, \ldots, 8$ (expected targets k killed during weather ℓ cannot exceed the available); $\sum_{i=1}^{10} c_i y_i + \sum_{j=1}^{15} d_j z_j \leq 100$ (total procurement of aircraft and munitions within budget); all variables nonnegative

4-40. max $\sum_{t=1}^{13} (-p w_t + \sum_{i=0}^{9} (s_i x_{i,t} + a_i y_{i,t} - r_i z_{i,t}))$ (max total sales + allowances - purchases - repurchases), s.t. $x_{0,t} + y_{0,t} \leq w_t$, $t = 1, \ldots, 13$ (sales of new tractors in time period t at most new purchases); $\sum_{i=0}^{9} y_{i,t} \leq w_t$, $t = 1, \ldots, 13$ (tradeins of tractors in time period t at most new purchases); $f_{i,t} + x_{i,t} - z_{i,t} = f_{i,t+1}$, $i = 0, \ldots, 9$, $t = 1, \ldots, 12$ (tractor balance of age i in time period t); $f_{i,13} + x_{i,13} - z_{i,13} = f_{i+1,1}$, $i = 0, \ldots, 8$ (tractor balance of age i in ending time period 13); $f_{9,13} + x_{9,13} - z_{9,13} = 0$; $\ell_t \leq \sum_{i=0}^{9} f_{i,t} \leq u_t$, $t = 1, \ldots, 13$ (fleet size in period t between min and max limits); all variables nonnegative