

# Chapter 1

## Exercise Solutions

1-1. (a) The only unsettled quantity is decision variable  $s$ . (b) Given quantities or parameters are  $d$ ,  $p$  and  $b$ . (c) Minimize the maximum error, i.e. objective  $\min (d/s)^2$  (d) We must have an integer number of sensors and not exceed the available budget, i.e. constraints  $ps \leq b$ ,  $s$  non-negative and integer.

1-2. (a) Feasible because  $3.5(4) \leq 14$ , and optimal because any larger  $s$  would not be feasible. (b) Infeasible and thus not optimal because  $3.5(6) \not\leq 14$ . (c) Feasible because  $3.5(2) \leq 14$ , but not optimal because feasible solution  $s = 4$  yields a better objective value.

1-3. (a) The only quantities to be determined are  $x_1$  and  $x_2$ , the numbers of lots on the 2 lines. (b) Given quantities or parameters are  $t_1$ ,  $t_2$ ,  $c_1$ ,  $c_2$ ,  $b$  and  $T$ . (c) Minimize total production cost or objective  $\min c_1x_1 + c_2x_2$ . (d)  $t_1x_1 + t_2x_2 \leq T$  (at most  $T$  hours of production),  $x_1 + x_2 = b$  (produce  $b$  lots),  $x_1, x_2 \geq 0$  and integer (numbers nonnegative integers).

1-4. (a) Infeasible and thus not optimal because  $10(0) + 20(3) \not\leq 40$ . (b) Feasible because  $10(2) + 20(1) \leq 40$  and  $2 + 1 = 3$ . Also optimal because no more or less expensive  $x_2$  can be used if  $b = 3$  lots are to run. (c) Feasible because  $10(3) + 20(0) \leq 40$  and  $3 + 0 = 3$ , but not optimal because  $x_1 = 2$ ,  $x_2 = 1$  yields a lower cost.

1-5. (a) Exact numerical optimization because it is the maximum feasible choice for the given set of parameter values. (b) Descriptive modeling because we have merely evaluated the consequences of a given choice of decision variables and parameters. (c) Closed-form optimization

because an optimal solution is specified for each choice of decision variables. (d) Heuristic optimization because a good feasible solution is identified for the given choice of parameter values, but a non-usual layout might yield superior results.

1-6. (a) Provides optimum for all choices of input parameters, not just one. (b) Provides a provably best solution, not just a good feasible one. (c) Systematically searches for a good feasible solution, rather than just evaluating the consequences of one.

1-7. Higher tractability usually means loss of validity, so results from the model might not be useful in the application.

1-8. (a) (3 for the first)  $\cdot$  (3 for the second)  $\cdot \dots \cdot$  (3 for the  $n$ th)  $= 3^n$  combinations. (b) One run per second is 3,600 per hour, 86,400 per day, 31,536,000 per year. The  $3^{10} = 59,049$  requires  $59,049/3,600 = 16.4$  hours;  $3^{15} = 14,348,907$  requires 166.1 days;  $3^{20} \approx 3.49 \times 10^9$  requires 110.6 years; and  $3^{30} \approx 2.06 \times 10^{14}$  requires 6.5 million years. (c) Practical computation would be limited to a few days which could accommodate no more than 10 – 11 decision variables.

1-9. (a) Random variable because short term rainfall is unpredictable. (b) Deterministic quantity because annual rainfall averages are fairly stable. (c) Deterministic quantity because history can be known with certainty. (d) Random variable because future stock market behavior is highly uncertain. (e) Deterministic quantity because the seating capacity is fairly fixed. (f) Random variable because night to night arrivals are usually variable. (g) Random variable

because breakdowns make the effective production rate uncertain. (h) Deterministic quantity because a reliable robot has a predictable rate of production. (i) Deterministic quantity because short term demand for such an expensive product would be fairly well known for the next few days. (j) Random variable because long term demand for a product is usually uncertain.