A

Name:

Multivariable Calculus and Matrix Algebra, MATH2010

Second Exam, Wednesday, October 8, 2003.

You may use one sheet of handwritten notes, but no other sources. The exam consists of four questions, and lasts fifty minutes. Please work all three problems. Please show all work clearly and in reasonable detail. Answers without appropriate supporting work or requested explanations may not receive full credit. No calculators are allowed.

Please ring your section below.

7: 11am	6: 1pm	8: 3pm	5: 4pm
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SOLUTIONS.

Q1	
Q2	
Q3	
Q4	
Total	
Grade	

1. (25 points) Find the arc length of the plane curve C given by $\mathbf{r}(t) = 2t^{3/2}\mathbf{i} + 4t\mathbf{j}$, $0 \le t \le 1$.

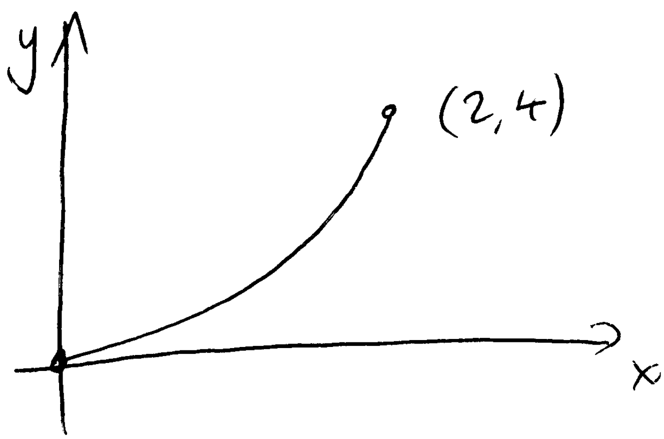
$$\text{Length} = \int_C 1\, ds = \int_{t=0}^{1} 1\, |r'(t)|\, dt$$

$$= \int_0^1 \sqrt{(3t^{\frac{1}{2}})^2 + 4^2}\, dt = \int_0^1 \sqrt{9t+16}\, dt$$

$$= \frac{1}{9}\frac{2}{3}\Big[(9t+16)^{3/2}\Big]_0^1$$

$$= \frac{2}{27}(125-64)$$

$$= \frac{122}{27}.$$



2. (25 points) The vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$. The curve C is given by $\mathbf{r}(t) = 2\cos t\mathbf{i} + 3\sin t\mathbf{j}$, $0 \le t \le \frac{\pi}{2}$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

$$\int_C \underline{F} \cdot d\underline{r} = \int_C \underline{F} \cdot \frac{d\underline{r}}{dt}\, dt$$

$$= \int_0^{\pi/2} \left((-3\sin t)(-2\sin t) + (2\cos t)(3\cos t) \right) dt$$

$$= 6 \int_0^{\pi/2} \sin^2 t + \cos^2 t \, dt$$

$$= 6 \int_0^{\pi/2} 1 \, dt$$

$$= \frac{6\pi}{2}$$

$$= \boxed{3\pi}$$

3. (25 points) Let $f(x, y, z) = x^2y + 2yz + 4xz$, and define the vector field $\mathbf{F} = \nabla f = (2xy + 4z)\mathbf{i} + (x^2 + 2z)\mathbf{j} + (2y + 4x)\mathbf{k}$.

   (a) (15 points) Let C be the curve given by $\mathbf{r}(t) = \sin(\frac{\pi t}{2})\mathbf{i} + (2^t - 1)\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

   (b) (10 points) What is curl ($\mathbf{F}$)? Justify your answer.

(a) Since $\underline{F} = \nabla f$, we know $\underline{F}$ is conservative.

Thus, $\int_C \underline{F} \cdot d\underline{r} = f(\underline{r}(1)) - f(\underline{r}(0))$

$= f(1, 1, 1) - f(0, 0, 0)$

$= 7 - 0$

$= \boxed{7}$

(b) Since $\underline{F}$ is conservative, $\boxed{\text{curl}(\underline{F}) = \underline{0}}$

4. (25 points) Let C be the circle of radius 2 centered at the origin, traversed in a counterclockwise direction. Show that $\oint_C y^2dx - x^2dy = 0$.

Does it follow that $\mathbf{F} = y^2\mathbf{i} - x^2\mathbf{j}$ is conservative? Justify your answer.

By Green's Thm,

$$\oint_C y^2dx - x^2dy = \iint_D -2x-2y\,dA$$

where D = area inside the circle

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2}(-2r\sin\theta - 2r\cos\theta)\,r\,dr\,d\theta$$

in polar coordinates

$$= -\int_{\theta=0}^{2\pi}(\sin\theta + \cos\theta)\left[\frac{2}{3}r^3\right]_0^2 d\theta$$

$$= -\frac{16}{3}\left[-\cos\theta + \sin\theta\right]_0^{2\pi}$$

$$= -\frac{16}{3}\left[-1+0+1-0\right]$$

$$= \boxed{0}$$

It does not follow that F is conservative

Need $\oint_C F\cdot dr = 0$ for every closed smooth curve C for this.

Also, $-\frac{\partial}{\partial y}(+y^2) + \frac{\partial}{\partial x}(\cancel{y}-x^2) = -2(x+y)$, which is not identically zero.