Birge & Louveaux, page 18, Question 2.

10% less than average prices

$$\min \; 150x_1 + 230x_2 + 260x_3$$

$$-\tfrac{1}{3}\left(153w_{11} - 214y_{11} + 135w_{21} - 189y_{21} + 36w_{31} + 10w_{41}\right)$$

$$-\tfrac{1}{3}\left(170w_{12} - 238y_{12} + 150w_{22} - 210y_{22} + 36w_{32} + 10w_{42}\right)$$

$$-\tfrac{1}{3}\left(187w_{13} - 262y_{13} + 165w_{23} - 231y_{23} + 36w_{33} + 10w_{43}\right)$$

s.t. Constraints exactly as in equation (1.2) on page 8.

10% greater than average prices.

Birge & Louveaux, Page 18, Question 3.

Let $x_{ij} = \begin{cases} 1 & \text{if crop } i \text{ planted in field } j \\ 0 & \text{otherwise.} \end{cases}$

$i=1$:	wheat	$j=1$:	185 acres
2:	corn	2:	145 acres
3:	sugarbeets	3:	105 acres
		4:	65 acres.

~~Let~~
We will still use all the variables in eqn (1.2), and also use ~~x_i~~ the twelve x_{ij} variables.

Formulation is (1.2) plus the following additional constraints:

$$\left.\begin{aligned} x_1 &= 185x_{11} + 145x_{12} + 105x_{13} + 65x_{14} \\ x_2 &= 185x_{21} + 145x_{22} + 105x_{23} + 65x_{24} \\ x_3 &= 185x_{31} + 145x_{32} + 105x_{33} + 65x_{34} \end{aligned}\right\} \text{Calculate amount of each crop planted.}$$

x_{ij} binary, $i=1,2,3$, $j=1,\ldots,4$.

$$\left.\begin{aligned} x_{11} + x_{21} + x_{31} &= 1 \\ x_{12} + x_{22} + x_{32} &= 1 \\ x_{13} + x_{23} + x_{33} &= 1 \\ x_{14} + x_{24} + x_{34} &= 1 \end{aligned}\right\} \text{No more than one crop can be planted in any field.}$$

Birge & Louveaux, Page 18, Question 4.

Let z_{ij}^B = number of contracts of crop i bought in scenario j

z_{ij}^S = number of contracts of crop i sold in scenario j

$i=1$: wheat $\quad j=1$: above average

2: corn $\quad 2$: average

3: below average.

Recourse cost is expressed in terms of z_{ij}.

Need $y_{ij} \le 100 z_{ij}^B$, $\quad w_{ij} \ge 100 z_{ij}^S$

Can't give cows more than you buy $\quad$ Can't sell more than the excess crop.

$$\min 150x_1 + 230x_2 + 260x_3$$

$$-\tfrac{1}{3}\left(17000 z_{11}^S - 23800 z_{11}^B + 15000 z_{21}^S - 21000 z_{21}^B + 36w_{31} + 10w_{41}\right)$$

$$-\tfrac{1}{3}\left(17000 z_{12}^S - 23800 z_{12}^B + 15000 z_{22}^S - 21000 z_{22}^B + 36w_{32} + 10w_{42}\right)$$

$$-\tfrac{1}{3}\left(17000 z_{13}^S - 23800 z_{13}^B + 15000 z_{23}^S - 21000 z_{23}^B + 36w_{33} + 10w_{43}\right)$$

subject to $x_1 + x_2 + x_3 \le 500$ and all old constraints from (1.2)

+ extra constraints:

$y_{11} \le 100 z_{11}^B$, $y_{12} \le 100 z_{12}^B$, $y_{13} \le 100 z_{13}^B$

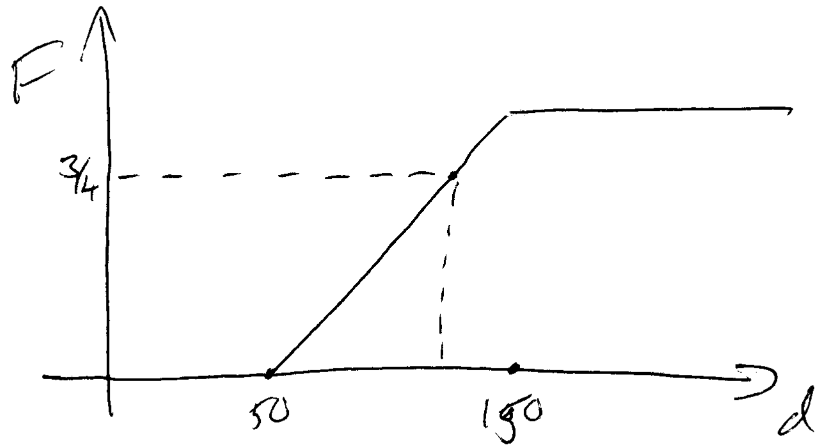
$y_{21} \le 100 z_{21}^B$, $y_{22} \le 100 z_{22}^B$, $y_{23} \le 100 z_{23}^B$

$w_{11} \ge 100 z_{11}^S$, $w_{12} \ge 100 z_{12}^S$, $w_{13} \ge 100 z_{13}^S$

$w_{21} \ge 100 z_{21}^B$, $w_{22} \ge 100 z_{22}^B$, $w_{23} \ge 100 z_{33}^B$

z_{ij}^B, z_{ij}^S binary, $i=1,2$, $j=1,2,3$.

BIRGE & LOUVEAUX, PAGE 20, QUESTION 10.



$$F(\xi) = \begin{cases} \dfrac{\xi - 50}{100} & \text{if } 50 \le \xi \le 150. \\ 1 & \text{if } \xi \ge 150 \\ 0 & \text{if } \xi \le 50. \end{cases}$$

$q = 25, \quad c = 10, \quad r = 5.$

So $\dfrac{q-c}{q-r} = \dfrac{15}{20} = \dfrac{3}{4}.$

$x = F^{-1}(3/4) = 50 + \frac{3}{4}(150 - 50) = \underline{\underline{125}}$ OPTIMAL SOLUTION.

Optimal value of stochastic program:

$$cx + \mathcal{Q}(x) = 1250 + \left(-qx + (q-r)\int_{50}^{x} F(\xi)\,d\xi\right) \quad \text{from page 17}$$

$$= 1250 - 3125 + 20\int_{50}^{125} \frac{\xi - 50}{100}\,d\xi$$

$$= -1875 + \frac{20}{100}\left[\frac{1}{2}\xi^2 - 50\xi\right]_{50}^{125}$$

$$= -1875 + \frac{1}{5}\left(\frac{1}{2}(15625) - 6250 - \frac{1}{2}(2500) + 2500\right)$$

$$= \underline{\underline{-1312.5}}.$$

Deterministic model:

Assume demand of 100.

So optimal solution is to buy 100 papers.

Cost is

$$100c + Q(100) = 1000 + \left(-100q + (q-r)\int_{50}^{100} F(\xi)\,d\xi\right)$$
$$= -1250.$$

The value of the stochastic solution is the difference between these two numbers:

$$1312.5 - 1250 = \boxed{62.5}$$

Birge & Louveaux, Page 28, Question 2

Having G vary can be modeled by introducing an extra stage before determining y and w.

Let $y(s_1, s_2, s_3, G)$ = surplus of returns follow scenarios s_1, s_2, s_3 and if final target is G.

$w(s_1, s_2, s_3, G)$ = deficit of returns follow scenarios s_1, s_2, s_3 and if final target is G.

Model:

$$\max \quad z = \frac{1}{16} \sum_{s_1=1}^{2} \sum_{s_2=1}^{2} \sum_{s_3=1}^{2} \sum_{\substack{G=75 \\ \text{or } 85}} y(s_1,s_2,s_3,G) - 4w(s_1,s_2,s_3,G)$$

$$\begin{aligned}
\text{s.t.} \quad & x(1,1) + x(2,1) = 55 \\
& -1.25x(1,1) - 1.14x(2,1) + x(1,2,1) + x(2,2,1) = 0 \\
& -1.06x(1,1) - 1.12x(2,1) + x(1,2,2) + x(2,2,2) = 0 \\
& -1.25x(1,2,1) - 1.14x(2,2,1) + x(1,3,1,1) + x(2,3,1,1) = 0 \\
& -1.06x(1,2,1) - 1.12x(2,2,1) + x(1,3,1,2) + x(2,3,1,2) = 0 \\
& -1.25x(1,2,2) - 1.14x(2,2,2) + x(1,3,2,1) + x(2,3,2,1) = 0 \\
& -1.06x(1,2,2) - 1.12x(2,2,2) + x(1,3,2,2) + x(2,3,2,2) = 0
\end{aligned}$$

$$\left.\begin{aligned}
& 1.25x(1,3,1,1) + 1.14x(2,3,1,1) - y(1,1,1,G) + w(1,1,1,G) = G \\
& 1.06x(1,3,1,1) + 1.12x(2,3,1,1) - y(1,1,2,G) + w(1,1,2,G) = G \\
& 1.25x(1,3,1,2) + 1.14x(1,3,1,2) - y(1,2,1,G) + w(1,2,1,G) = G \\
& 1.06x(1,3,1,2) + 1.12x(1,3,1,2) - y(1,2,2,G) + w(1,2,2,G) = G \\
& 1.25x(1,3,2,1) + 1.14x(1,3,2,1) - y(2,1,1,G) + w(2,1,1,G) = G \\
& 1.06x(1,3,2,1) + 1.12x(1,3,2,1) - y(2,1,2,G) + w(2,1,2,G) = G \\
& 1.25x(1,3,2,2) + 1.14x(1,3,2,2) - y(2,2,1,G) + w(2,2,1,G) = G \\
& 1.06x(1,3,2,2) + 1.12x(1,3,2,2) - y(2,2,2,G) + w(2,2,2,G) = G
\end{aligned}\right\} \begin{array}{l} G = 75 \\ \text{and } G = 85 \end{array}$$

$$x, y, w \geq 0.$$

Birge & Louveaux, Page 43, Question 1

Let $\begin{pmatrix} x_f \\ x_b \\ x_e \end{pmatrix}$ = number of seats partitioned into $\begin{pmatrix} \text{first} \\ \text{business} \\ \text{economy} \end{pmatrix}$ class

Let y_{cs} = number of seats of class c sold in scenario s.

Have three equally likely scenarios. Want to maximize expected revenue.

Model: $\max \frac{1}{3} \sum_{s=1}^{3} 3 y_{fs} + 2 y_{bs} + y_{es}$

$$\text{s.t. } 2x_f + 1.5x_b + x_e \leq 200$$

limit on number of seats on plane

$$y_{fs} \leq x_f, \quad y_{bs} \leq x_b, \quad y_{es} \leq x_e \qquad s = 1, 2, 3$$

Can't sell more tickets than we have seats.

$$y_{f1} \leq 20, \quad y_{f2} \leq 10, \quad y_{f3} \leq 5$$

Can't sell more first class tickets than the demand

$$y_{b1} \leq 50, \quad y_{b2} \leq 25, \quad y_{b3} \leq 10$$

Can't sell more business class tickets than the demand

$$y_{e1} \leq 200, \quad y_{e2} \leq 175, \quad y_{e3} \leq 150$$

Can't sell more economy class tickets than the demand.

$$x \geq 0, \quad y \geq 0.$$

Could also impose integrality restrictions on the variables.
Will assume we can round LP solution to a good enough integer solution.