A Comparison of Computational Efficiency and Accuracy in Single Neuron Models

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The Neuron

- **Dendrites**
- **Axon**
- **Soma**
- **K^+**, **Na^+**, **Cl^−**, **Ca^{++}**

Diagram shows a neuron with dendrites on one side and the axon on the other, with ions such as K^+, Na^+, Cl^−, and Ca^{++} indicated.
Hodgkin-Huxley Model (HH)

\[ C \frac{dV}{dt} = -I_L - I_{Na} - I_K - I_M - I_{syn} + I_{ext} \]

\[ I_L = g_L(V - E_L) \quad I_M = g_M n_M (V - E_K) \]

\[ I_{Na} = g_{Na} m^3 h (V - E_{Na}) \quad I_{syn} = g_E(t)(V - E_E) \]

\[ I_K = g_K n^4 (V - E_K) \quad I_{ext} = \text{External Current} \]
Hodgkin-Huxley Graph

HH Model with AMPA Input

Voltage (mV)

Time (ms)
Leaky Integrate & Fire (LIF)

\[
C \frac{dV}{dt} = -g_L (V - E_L) - g_E(t)(V - E_E)
\]
Exponential Integrate & Fire (EIF)

\[ C \frac{dV}{dt} = -g_L(V - E_L) - g_E(t)(V - E_E) + g_L \Delta T e^{\left(\frac{V - V_T}{\Delta T}\right)} \]
Adaptive EIF (aEIF)

\[ C \frac{dV}{dt} = -g_L(V - E_L) - g_E(t)(V - E_E) - g_L \Delta_T e^{\left( \frac{V - V_T}{\Delta_T} \right)} - w \]

\[ \tau_w \frac{dw}{dt} = a(V - E_L) - w \]

At firing time:

\[ V \rightarrow V_R \]

\[ w \rightarrow w + b \]
aEIF Graph

aEIF Model with AMPA Input

Voltage (mV)

Time (ms)
Hodgkin-Huxley vs. aEIF Model

Adaptive HH vs aEIF model

- Voltage (mV)
- Time (ms)

- aHH
- aEIF
Results

<table>
<thead>
<tr>
<th>Step sizes:</th>
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</thead>
<tbody>
<tr>
<td>• HH: $\Delta t=0.02\text{ms}$</td>
</tr>
<tr>
<td>• Adaptive HH: $\Delta t=0.16\text{ms},0.02\text{ms}$</td>
</tr>
<tr>
<td>• aEIF: $\Delta t=0.1\text{ms}$</td>
</tr>
<tr>
<td>• LIF: $\Delta t=0.1\text{ms}$</td>
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</tbody>
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| The accuracy is defined by the percentage of the number of sub-threshold points within 0.5 mV of HH. |

| Computational cost is defined as the method’s run-time divided by HH runtime. |

<table>
<thead>
<tr>
<th><strong>Accuracy</strong></th>
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<tr>
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Extra slides for questions
HH Detailed Equations

\[ I_L = g_L (V - E_L) \]

\[ g_L = 0.029 \mu S \]

\[ E_L = -70mV \]

\[ C = 0.29nF \]
HH Detailed Equations

\[ I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na}) \]

\[ \bar{g}_{Na} = 14.5 \mu S \quad E_{Na} = 55 mV \]

\[ m_\infty = \frac{\alpha_m(V)}{\alpha_m(V) + \beta_m(V)} \]

\[ \alpha_m(V) = \frac{-0.32(V-V_T-13)}{\exp(-(V-V_T-13)/4) - 1} \]

\[ \alpha_h(V) = 0.128 \exp(-(V-V_T-V_S-17)/18) \]

\[ h_\infty = \frac{\alpha_h(V)}{\alpha_h(V) + \beta_h(V)} \]

\[ \beta_m(V) = \frac{0.28(V-V_T-40)}{\exp((V-V_T-40)/5) - 1} \]

\[ \beta_h(V) = \frac{4}{1 + \exp(-)(V-V_T-V_S-40)/5} \]

\[ \tau_m = \frac{1}{\alpha_m(V) + \beta_m(V)} \]

\[ \tau_h = \frac{1}{\alpha_h(V) + \beta_h(V)} \]

\[ \frac{dm}{dt} = \frac{m_\infty - m}{\tau_m} \]

\[ \frac{dh}{dt} = \frac{h_\infty - h}{\tau_h} \]
HH Detailed Equations

\[ I_K = \bar{g}_K n^4 (V - E_K) \quad \bar{g}_K = 1.8 \mu S \quad E_K = -90 mV \]

\[ n_\infty = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_m(V)} \quad \tau_n = \frac{1}{\alpha_n(V) + \beta_n(V)} \]

\[ \alpha_n(V) = \frac{-0.032(V - V_T - 15)}{\exp(-(V - V_T - 15)) - 1} \]

\[ \beta_n(V) = 0.5 \exp(-(V - V_T - 10) / 40) \]
HH Detailed Equations

\[ I_M = \bar{g}_M n_M (V - E_K) \quad \bar{g}_M = 0.0203 \mu S \]

\[ n_\infty = \frac{\alpha_M(V)}{\alpha_M(V) + \beta_M(V)} \quad \tau_n = \frac{1/3}{\alpha_n(V) + \beta_n(V)} \]

\[ \alpha_M(V) = \frac{0.0001(V + 30)}{1 - \exp(-(V + 30)/9)} \]

\[ \beta_M(V) = \frac{-0.0001(V + 30)}{1 - \exp((V + 30)/9)} \]
Adaptive Time-stepping HH

- Use $\Delta t = 0.17$ for sub-threshold time-step.

- At slope cutoff of $|3|$ mV/ms, we use $\Delta t = 0.0253$ in order to resolve the spike.

The red stars represent where the slope is $|3|$ mV/ms and the size of the time step changes.