A Comparison of Computational Efficiency and Accuracy in Single Neuron Models

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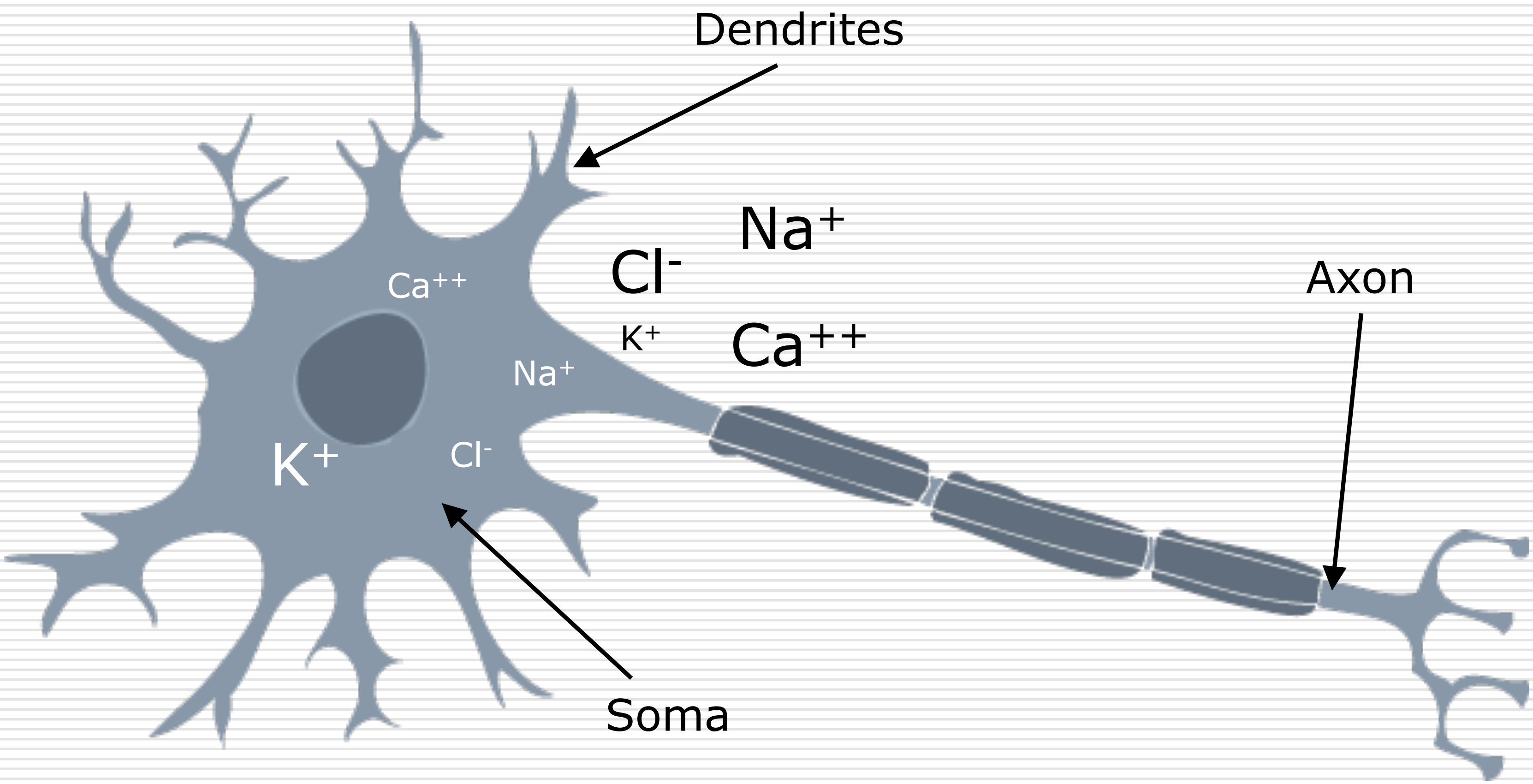
The Neuron



Hodgkin-Huxley Model (HH)

$$C\frac{dV}{dt} = -I_L - I_{Na} - I_K - I_M - I_{syn} + I_{ext}$$

$$I_L = g_L(V - E_L)$$

$$I_{Na} = \overline{g}_{Na} m^3 h(V - E_{Na})$$

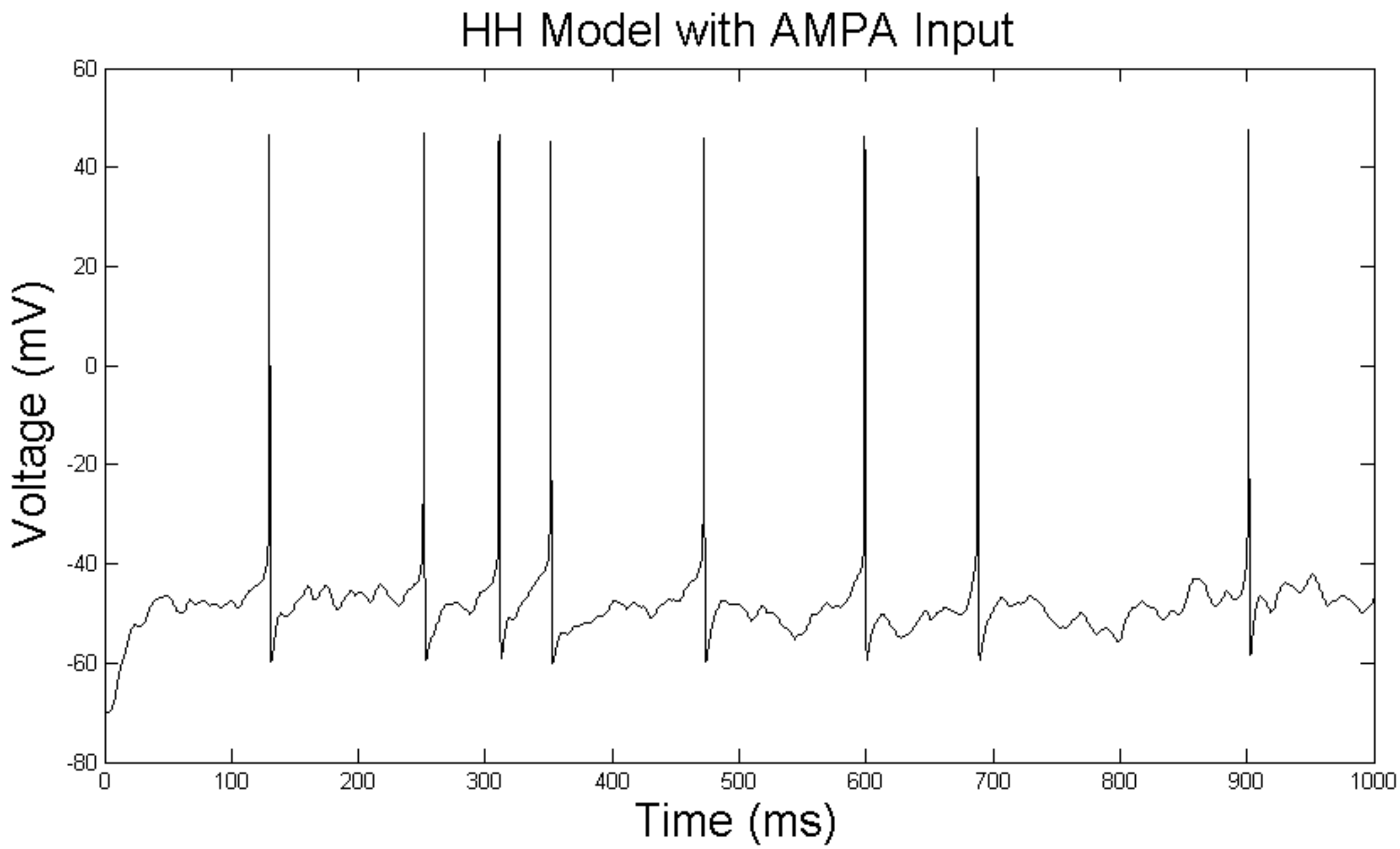
$$I_K = \overline{g}_K n^4 (V - E_K)$$

$$I_M = \overline{g}_M n_M (V - E_K)$$

$$I_{syn} = g_E(t)(V - E_E)$$

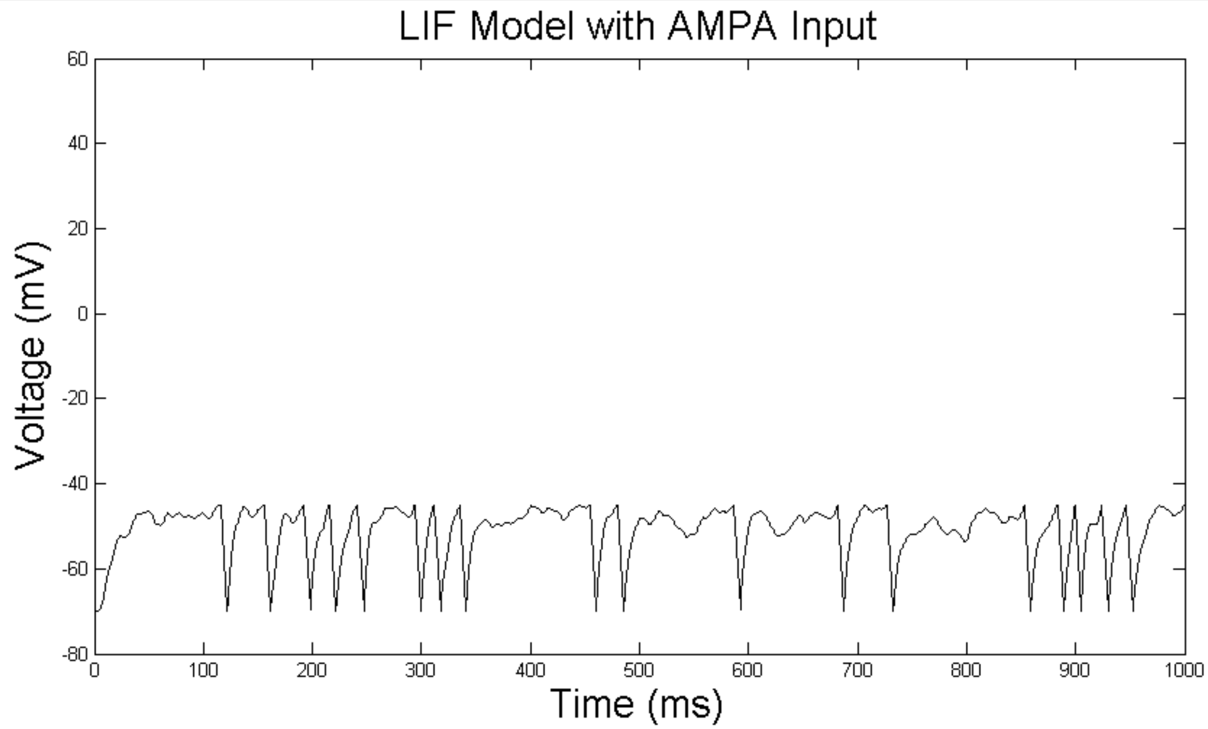
$$I_{ext} = External\,Current$$

Hodgkin-Huxley Graph



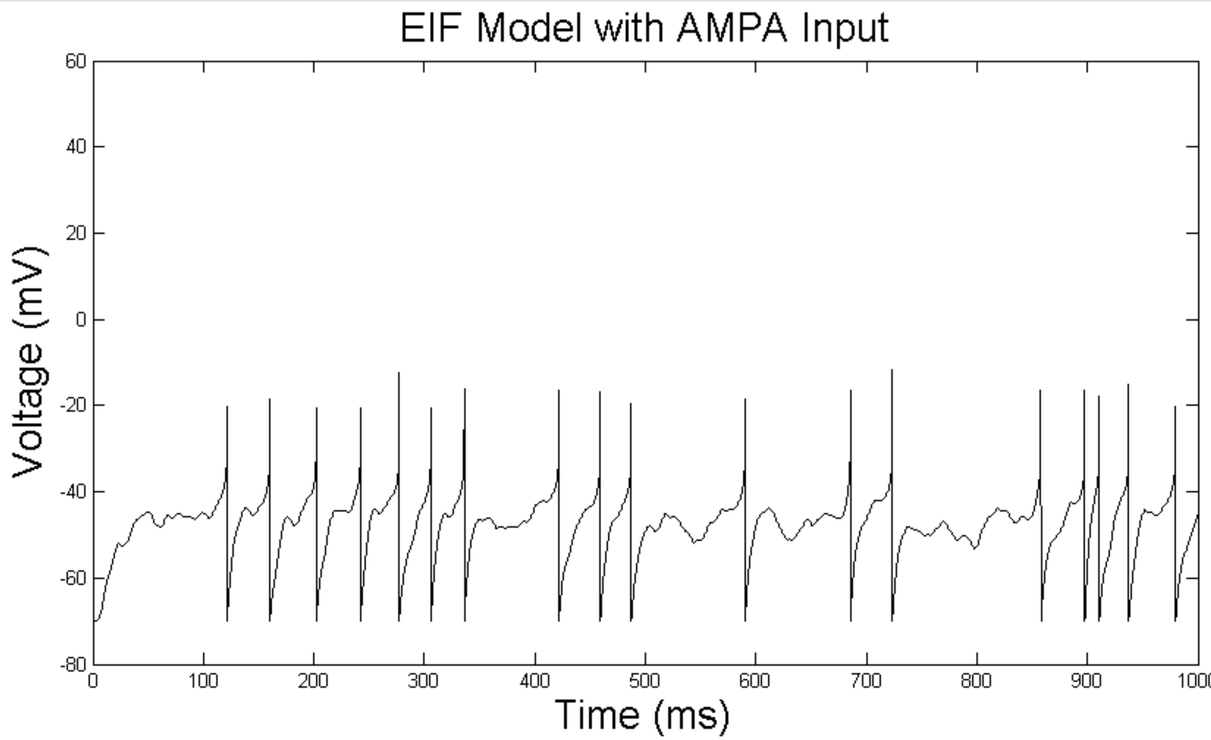
Leaky Integrate & Fire (LIF)

$$C\frac{dV}{dt} = -g_L\left(V - E_L\right) - g_E(t)\left(V - E_E\right)$$



Exponential Integrate & Fire (EIF)

$$C\frac{dV}{dt} = -g_L(V - E_L) - g_E(t)(V - E_E) + g_L\Delta_T e^{\left(\frac{V - V_T}{\Delta_T}\right)}$$



Adaptive EIF (aEIF)

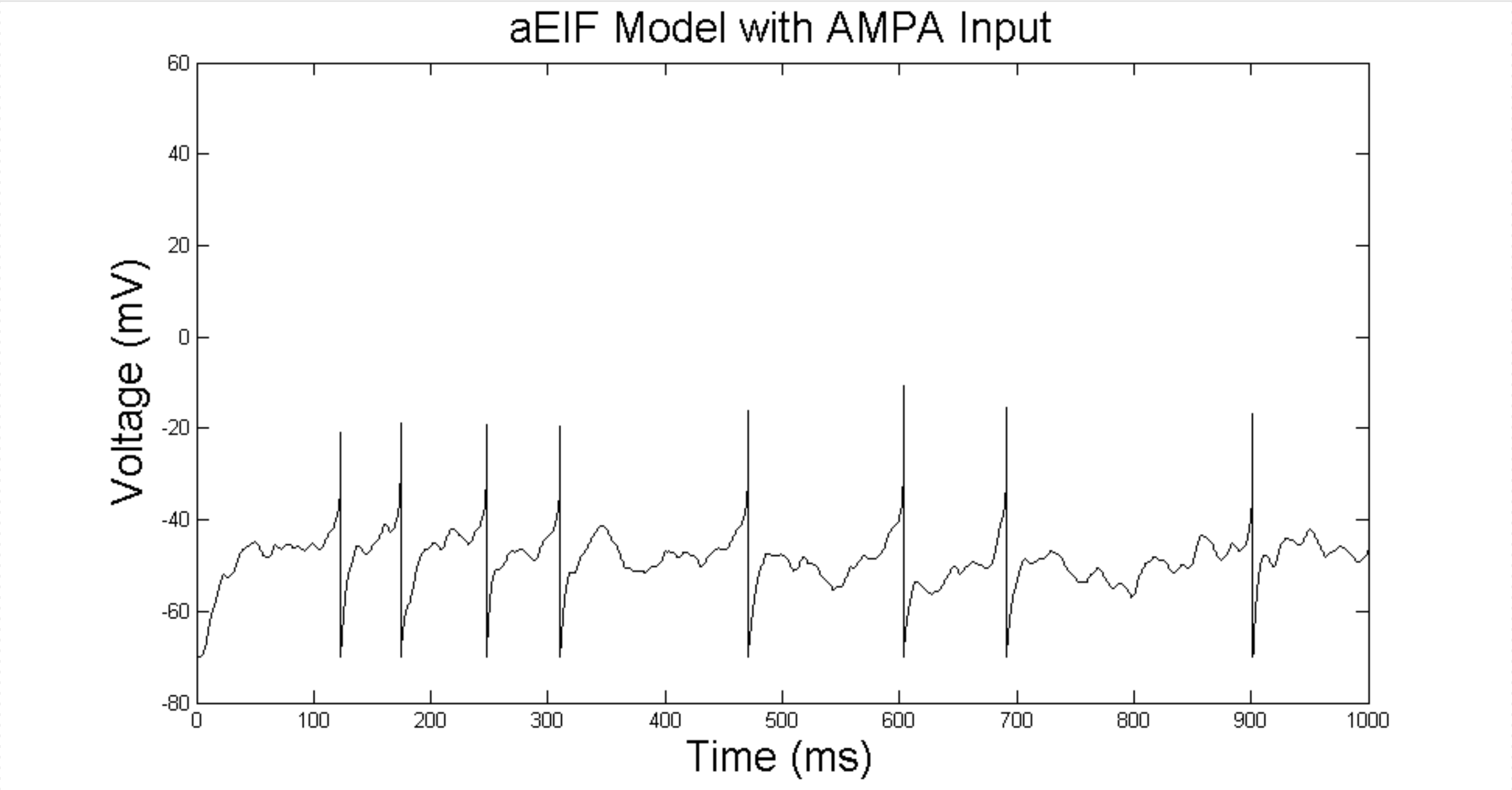
$$C\frac{dV}{dt} = -g_L\left(V - E_L\right) - g_E(t)\left(V - E_E\right) - g_L \Delta_T e^{\left(\frac{V - V_T}{\Delta_T}\right)} - w$$

$$\tau_w \frac{dw}{dt} = \mathrm{a}\left(V - E_L\right) - w$$

At firing time:

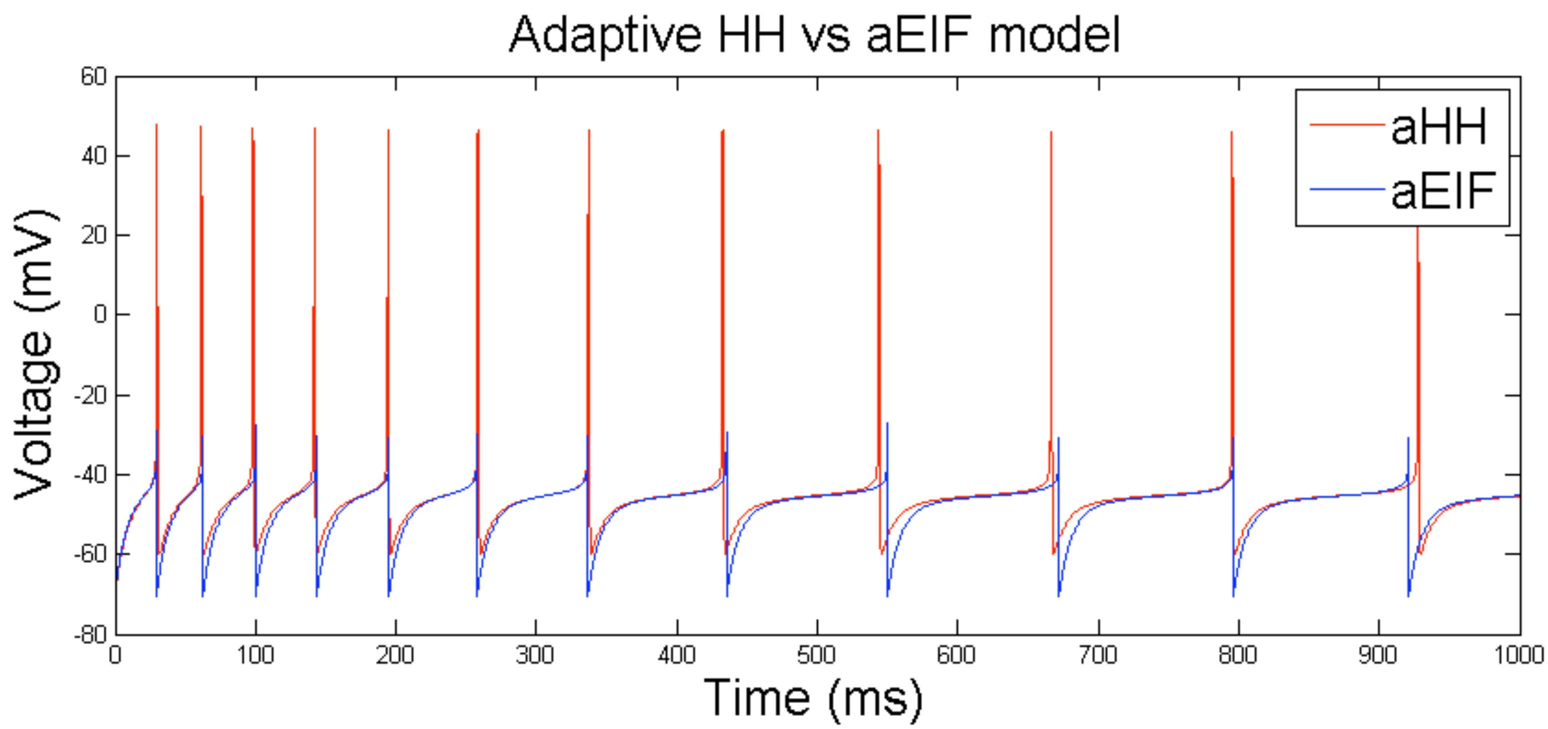
$$V \to V_R$$
$$w \to w + b$$

aEIF Graph



aEIF Model with AMPA Input
60
40
20
0
-20
-40
-60
-80
0
100
200
300
400
500
600
700
800
900
1000
Voltage (mV)
Time (ms)

Hodgkin-Huxley vs. aEIF Model



Results

- Step sizes:
  - HH: Δt=0.02ms
  - Adaptive HH:Δt=0.16ms,0.02ms
  - aEIF : Δt=0.1ms
  - LIF: Δt=0.1ms

- The accuracy is defined by the percentage of the number of sub-threshold points within 0.5 mV of HH.

- Computational cost is defined as the method's run-time divided by HH run-time.

Accuracy	
HH	1.000
Adaptive HH	0.997
aEIF	0.801
LIF	0.275

Computational Cost	
HH	1.000
Adaptive HH	0.195
aEIF	0.130
LIF	0.055

Extra slides for questions

HH Detailed Equations

$$I_L = \overline{g}_L (V - E_L)$$

$$g_L = 0.029 \mu S$$

$$E_L = -70 mV$$

$$C = 0.29 nF$$

HH Detailed Equations

$$I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na})$$

$$\bar{g}_{Na} = 14.5 \mu S \qquad E_{Na} = 55 mV$$

$$m_\infty = \frac{\alpha_m(V)}{\alpha_m(V) + \beta_m(V)}$$

$$h_\infty = \frac{\alpha_h(V)}{\alpha_h(V) + \beta_h(V)}$$

$$\alpha_m(V) = \frac{-0.32(V - V_T - 13)}{\exp(-(V - V_T - 13)/4) - 1}$$

$$\alpha_h(V) = 0.128 \exp(-(V - V_T - V_S - 17)/18)$$

$$\beta_m(V) = \frac{0.28(V - V_T - 40)}{\exp((V - V_T - 40)/5) - 1}$$

$$\beta_h(V) = \frac{4}{1 + \exp(-)V - V_T - V_S - 40)/5)}$$

$$\tau_m = \frac{1}{\alpha_m(V) + \beta_m(V)}$$

$$\tau_h = \frac{1}{\alpha_h(V) + \beta_h(V)}$$

$$\frac{dm}{dt} = \frac{m_\infty - m}{\tau_m}$$

$$\frac{dh}{dt} = \frac{h_\infty - h}{\tau_h}$$

HH Detailed Equations

$$I_K = \overline{g}_K n^4 \left(V - E_K\right) \qquad \overline{g}_K = 1.8\mu S \qquad E_K = -90mV$$

$$n_\infty = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_m(V)} \qquad \tau_n = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

$$\alpha_n(V) = \frac{-0.032(V - V_T - 15)}{\exp(-(V - V_T - 15)) - 1} \qquad \frac{dn}{dt} = \frac{n_\infty - n}{\tau_n}$$

$$\beta_n(V) = 0.5\exp(-(V - V_T - 10)/40)$$

HH Detailed Equations

$$I_M = \bar{g}_M n_M (V - E_K) \quad \bar{g}_M = 0.0203 \mu S$$

$$n_\infty = \frac{\alpha_M(V)}{\alpha_M(V) + \beta_M(V)} \qquad \tau_n = \frac{1/3}{\alpha_n(V) + \beta_n(V)}$$

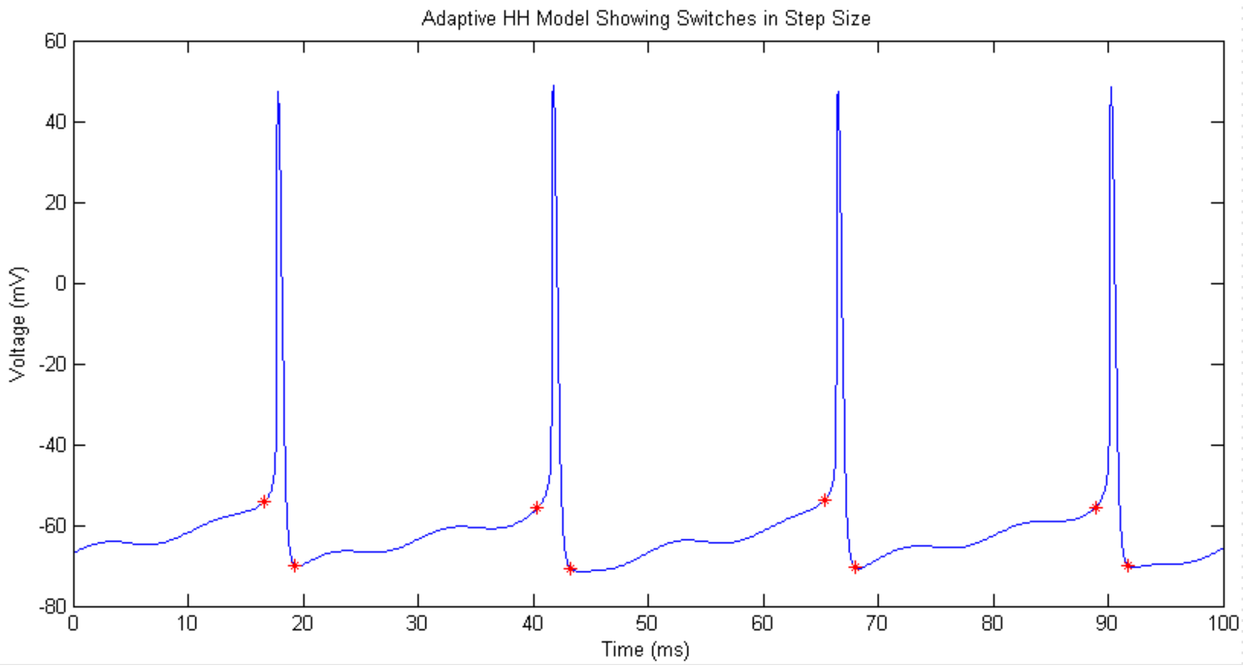
$$\alpha_M(V) = \frac{0.0001(V+30)}{1 - \exp(-(V+30)/9)} \qquad \frac{dn}{dt} = \frac{n_\infty - n}{\tau_n}$$

$$\beta_M(V) = \frac{-0.0001(V+30)}{1 - \exp((V+30)/9)}$$

Adaptive Time-stepping HH

- Use Δt=0.17 for sub-threshold time-step.

•At slope cutoff of |3| mV/ms, we use Δt=0.0253 in order to resolve the spike.



The red stars represent where the slope is |3| mV/ms and the size of the time step changes.